On a unusual application of ODEs to solve linear systems

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We deal with the solution of the (possibly) ill-conditioned linear system

$$Ac = b \tag{1}$$

where $A \in \mathbb{R}^{N \times N}$ is a bounded linear operator in a Hilbert space H, $c \in \mathbb{R}^N$, $b \in \mathbb{R}^N$.

The method proposed in this paper is based on the approach called Dynamical Systems Method (DSM), suggested by A. Ramm in [1], and later, e.g. in [2].
The DSM method for solving (1) consists of solving the Cauchy problem

$$x'(t) = \Theta(t, x(t)), \quad x(0) = x_0$$

where $x_0 \in H$ is an arbitrary element of a Hilbert space H and Θ is a nonlinear function, chosen so that the following conditions hold: a) there exists a unique solution $x(t)$ $\forall t \geq 0$, b) there exists $x(\infty)$ such that $A(x(\infty)) = b$ and $x(\infty) = c$. Usually, the DSM method chooses $\Theta(t, x(t)) = (A^T A + \lambda(t)\, I)^{-1} A^T b - x(t)$. Actually, the DSM method swifts the search for the solution of (1) to the calculation of the solution of another problem, which reduces to a vectorial differential equation. Here the novelty consists in introducing a general regularization operator B, which replaces the identity regularization operator I in $\Theta(t, x(t))$, so that the vectorial differential equation to be solved becomes

$$x'(t) = -x(t) + (A^T A + \lambda(t)\, B^T B)^{-1}\, A^T b, \quad x(0) = x_0 \tag{2}$$

Convergence of solution of (2) to solution of (1) is proved. Then (2) is solved by a one-step explicit numerical method, which converges to the solution of (2) in a fast way. Therefore the overall method is characterized by a fast total convergence which achieves competitive value of relative errors in very few steps. Some enlightening numerical examples are presented.

References

[1] A.G. Ramm, *Dynamical Systems Method for solving operator equations*, Elsevier, Amsterdam-Boston; 2007

[2] S. Indratno, A.G. Ramm, Dynamical systems method for solving ill-conditioned linear algebraic systems, *Int. J. Comput. Sci. Math.* 2009; **2**: 308–333