Performance of "Look-Ahead" Linear Multistep Methods

Taketomo MITSUI*

Abstract

We are concerned with the initial-value problem of ordinary differential equations (ODEs):

$$\frac{\mathrm{d}y}{\mathrm{d}x} = f(x,y) \quad (a \le x \le b), \quad y(a) = y_I.$$

LALMM, which stands for "look-ahead" linear multistep methods, is a new class among the discrete variable methods (DVMs) for the problem. Its mechanism is as follows: Assume that we look for the numerical solution of the $(n+k)$-th step-point when the back-values $y_n, y_{n+1}, \ldots, y_{n+k-1}$ and a preassigned *initial guess* $y_{n+k}^{[0]}$ are available. First, we look ahead for the $(n+k+1)$-st step-point by

$$y_{n+k+1}^{[0]} + \alpha_k y_{n+k}^{[0]} + \sum_{i=0}^{k-1} \alpha_i y_{n+i} = h\left(\beta_k f(x_{n+k}, y_{n+k}^{[0]}) + \sum_{i=0}^{k-1} \beta_i f(x_{n+i}, y_{n+i})\right),$$

which can be regarded as a predictor. Then, correct the look-for value by

$$y_{n+k}^{[1]} + \sum_{i=0}^{k-1} \alpha_i^* y_{n+i} = h\left(\beta_{k+1}^* f(x_{n+k+1}, y_{n+k+1}^{[0]}) + \beta_k^* f(x_{n+k}, y_{n+k}^{[0]}) + \sum_{i=0}^{k-1} \beta_i^* f(x_{n+i}, y_{n+i})\right).$$

When a (local) convergence attains, *i.e.,* the estimation $\|y_{n+k}^{[1]} - y_{n+k}^{[0]}\| \le \delta_{TOL}$ holds for a pre-assigned error tolerance δ_{TOL}, we complete the current step and advance to the next step. Otherwise, we replace $y_{n+k}^{[0]}$ by $y_{n+k}^{[1]}$ and iterate prediction and correction. Note that we employ equi-distant step points $\{x_n\}$ and approximations $\{y_n\}$ on them.

The core issue of numerical analysis of new methods is whether they can perform better than the existing methods. We derived several LALMM schemes of two-step family (LALTM) and examine their performance through test examples of ODEs. We will report the test results of LALTMs by several numerical examples and describe a possible way to overcome their difficulties shown in the examples.

*Professor Emeritus, Nagoya University tom.mitsui@nagoya-u.jp