Wavelets based on Hermite cubic splines

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Abstract.

A general concept for solving of operator equations by means of wavelets was proposed by A. Cohen, W. Dahmen and R. DeVore in [7, 8]. It consists of the following steps: transformation of the variational formulation into the well-conditioned infinite-dimensional problem in the space l^2, finding of the convergent iteration process for the l^2- problem and finally a derivation of its computable version. The aim is to find an approximation of the unknown solution u which should correspond to the best N-term approximation, and the associated computational work should be proportional to the number of unknowns. Essential components to achieve this goal are well-conditioned wavelet stiffness matrices and an efficient approximate multiplication of quasi-sparse wavelet stiffness matrices with vectors.

In [7], authors exploited an off-diagonal decay of the entries of the wavelet stiffness matrices and designed a numerical routine **APPLY** which approximates the exact matrix-vector product with the desired tolerance ε and that has linear computational complexity, up to sorting operations. Although it has optimal computational complexity, its application is relatively time consuming and moreover it is not easy to implement it efficiently. Further it is well known, that the condition numbers of the stiffness matrices in wavelet coordinates depend on Riesz constants of a wavelet basis [1]. Thus we can conclude that it is useful to develop well-conditioned wavelet bases on the interval. Well-conditioned wavelet bases for different types of wavelets and for different types of boundary conditions were already constructed in [2, 3, 4, 5, 10]. In this talk, we introduce a construction of the wavelet basis based on Hermite cubic splines with respect to which both the mass matrix and the stiffness matrix corresponding to one dimensional Poisson equation are sparse. Then, matrix-vector multiplication can be performed exactly with linear complexity for any second order differential equation with constant coefficients. Wavelets with similar properties were already proposed in [6, 9]. Our wavelets generate the same multiresolution spaces as wavelets from [6, 9] but have improved condition numbers. In comparison with wavelets from [6], we constructed the fourth wavelet in a different way.

Keywords: Hermite cubic spline-wavelets, sparse representation, Riesz basis, preconditioning
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