Whispering gallery modes in oblate spheroidal cavities: calculations with a variable stepsize

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Abstract

The aim of this talk is to present the obtained progress in the numerical simulation of the so-called Whispering Gallery Modes (WGMs) occurring inside an oblate spheroidal cavity. The calculation of such modes is difficult since they are strongly confined within a very narrow region in the near-equatorial domain of the cavity.

Following the idea of separating variables in spheroidal coordinates proposed in [2, 3], the two ODEs related to the angular and radial coordinates form the singular self–adjoint two parameter Sturm–Liouville problem

$$\frac{d}{d\eta}(1-\eta^2)\frac{d}{d\eta}S + \left[\lambda - c^2(1-\eta^2) - \frac{m^2}{1-\eta^2}\right] S = 0, \qquad -1 < \eta < 1,$$
$$\frac{d}{d\xi}(\xi^2+1)\frac{d}{d\xi}R + \left[c^2(\xi^2+1) - \lambda + \frac{m^2}{\xi^2+1}\right] R = 0, \qquad 0 < \xi < \xi_s.$$

We propose an efficient and reliable approach combining the Prüfer angle technique, applied to provide a starting good approximation for the parameter pair (λ, c^2), and high order finite difference schemes with a variable stepsize [1] based on the error equidistribution, to reach high accuracy in the computation of the radial and angular functions and to improve the evaluation of (λ, c^2).

We illustrate the approach by numerical simulations concerning highly localized WGMs inside an oblate spheroidal cavity.

References

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