Efficient accurate non-iterative break point detection and computation for state-dependent delay differential equations

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Abstract

When solving delay differential equations (DDEs) with state-dependent delays the problem of breaking point detection becomes important. Those points where the solution is not smooth enough to provide the order of the method applied must be included into the mesh, so not to fall inside a step, which would result in a reduction of the order of the solution. The problem, however is to detect and compute such points efficiently. Breaking points arise every time a delay falls on a previous breaking point (either of the calculated solution or in the history function). In the case of retarded DDEs the new breaking point is (at least) one order smoother than the previous breaking point that gave rise to it. For fixed or time-dependent delays the breaking points can be precomputed independent of the solution, but for state-dependent delays the positions of the breaking points depend on the computed solution. If a breaking point is detected and the step-size is changed in order to incorporate the point into the mesh, then the new step-size generates a new solution and the breaking point moves. Consequently, break point detection is traditionally performed iteratively, and is computationally expensive. The same break point can also be detected multiple times. In the current work we propose a fast non-iterative method for finding breaking points with sufficient precision to preserve the order of up to third or fourth order methods. Our method makes use of analytic continuation of the solution across breaking points (including possible breaking points in the initial history function), and we explain how we handle this carefully to attain the desired order.