

Numerical infinitesimals for solving ODEs given as a Black-Box

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Abstract

We consider an initial value problem where the right-hand part of the ODE is given by a Black-Box computer procedure. Since all traditional computers work with finite numbers, only finite values for the integration step h can be used in numerical algorithms approximating the solution of the ODE. In this lecture, a new framework for solving ODEs is discussed for a new kind of a computer – the Infinity Computer – that has been patented recently (its working software prototype exists). The Infinity Computer is able to work numerically not only with finite quantities but also with infinities and infinitesimals. This fact gives the possibility to work with different infinitesimals numerically and, in particular, to take advantage of infinitesimal values of h . It is proved that the Infinity Computer is able to calculate derivatives of the solution $y(x)$ and to reconstruct its Taylor expansion of a desired order numerically. This is done automatically, without finding the respective derivatives analytically (or symbolically) by the successive derivation of the ODE as it is usually done when the Taylor method is applied.