Collocation for Singular BVPs in ODEs with Unsmooth Data

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We deal with BVPs for systems of ODEs with singularities. Typically, such problems have the form

$$z'(t) = \frac{M(t)}{t} z(t) + f(t, z(t)), \;\; t \in (0, 1], \quad B_0 z(0) + B_1 z(1) = \beta,$$

where B_0 and B_1 are constant matrices which are subject to certain restrictions for a well-posed problem. Here, we focus on the linear case where the function f is unsmooth, $f(t) = g(t)/t$. We first deal with the analytical properties of the problem – existence and uniqueness of smooth solutions. To solve the problem numerically, we apply polynomial collocation and for the linear IVPs, we are able to provide the convergence analysis. It turns out that the collocation retains its high order even in case of singularities, provided that the analytical solution is sufficiently smooth. We illustrate the theory by numerical experiments; the related tests were carried out using the Matlab code `sbvp` [1].

It also turns out that collocation can be applied to solve differential-algebraic equations of higher index. We illustrate this fact by means of numerical experiments.

References

[1] W. Auzinger, G. Kneisl, O. Koch, E.B. Weinmüller, A Collocation Code for Boundary Value Problems in Ordinary Differential Equations. *Numer. Algorithms, 33*, (2003), 27-39.