A Smooth Solution to a Nonlinear System of Singular ODEs

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Abstract

We examine the existence of C^m-smooth solutions to the $n \times n$-system of ODEs $tu'(t) = g(t, u(t))$, $0 \le t \le T$ with an m-smooth vector function $g(t, u)$ and m-smooth Jacobi matrix $\partial g(t, u)/\partial u$. Let $\omega_0 \in \mathbf{R}^n$ be a solution of the $n \times n$-system $g(0, \omega) = 0$; denote $A_0 = [\partial g(0, u)/\partial u]_{u=\omega_0}$ and assume that $m > \max\{\mathrm{Re}\lambda_k : \lambda_k \in \sigma(A_0)\}$. If $\sigma(A_0) \cap \mathbf{N}$ is void then there exists a unique C^m-smooth solutions such that $u(0) = \omega_0$. In the case of non-void $\sigma(A_0) \cap \mathbf{N}$ we describe consistency conditions that are necessary and sufficient for the existence of a C^m-smooth solution; under a suitable initial conditions it is unique. We propose a spline collocation method of accuracy $O(h^m |\log h|)$ for the problem.

Keywords: Singular differential equations, smooth solutions, spline collocation.

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