The construction of well-conditioned wavelet basis based on quadratic B-splines

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Abstract

The design of most adaptive wavelet methods for solving differential equations follows a general concept proposed by A. Cohen, W. Dahmen, and R. DeVore in [1, 2]. The essential steps are: to transform the variational formulation into the well-conditioned infinite-dimensional l^2-problem, to find the convergent iteration process for this infinite-dimensional l^2-problem and finally to derive its finite-dimensional approximation which works with an inexact right hand-side and an approximate matrix-vector multiplication. It should provide an approximation of the unknown solution up to a given target accuracy ϵ. To perform this scheme efficiently, it is necessary to have at one's disposal suitable wavelet bases which fit well into this concept. Wavelets should have short supports and vanishing moments, be smooth and known in closed form, and a corresponding wavelet basis should be well-conditioned. In our contribution, we propose a quadratic wavelet basis adapted to the interval $[0, 1]$ which preserves vanishing moments. Moreover, boundary wavelets are orthogonal to the first inner wavelets.

References

[1] Cohen, A., Dahmen, W. and DeVore, R.: *Adaptive Wavelet Schemes for Elliptic Operator Equations - Convergence Rates*, *Math. Comput.* **70**, no. 233, 27-75, 2001.

[2] Cohen, A., Dahmen, W. and DeVore, R.: *Adaptive Wavelet Methods II - Beyond the elliptic case*, *Found. Math.* **2**, no. 3, 203-245, 2002.