A parallel implementation of the Gauss method for ODEs

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Our starting point is the following result [1]:

Any Runge-Kutta collocation method admits an infinite family of equivalent representations, each defined by a (different) Butcher tableau with rank deficient matrix.

We elucidate the above result by considering low-rank versions of the Gauss collocation method.

One appealing point that make the study of such variants interesting is that on the one hand, they define the very same numerical solution of the classical Gauss method at each step of the integration procedure and, on the other hand, they open new routes of investigations for what concerns their implementation on a computer.

In particular, we here consider a diagonal splitting of the Butcher matrix and show that formulae in this family exist such that the corresponding nonlinear iteration, used to solve the associated nonlinear systems, be A-convergent.

Since the splitting is diagonal, the iteration may be carried out in parallel by constructing a number of processes equal to the number of stages of the underlying Gauss method (parallelization through the method).

We show some preliminary tests on (low-rank) Gauss methods of order 4, 6 and 8 and comment on the obtained results.

References

[1] L. Brugnano and F. Iavernaro, A low-rank representation of Runge–Kutta collocation methods, in preparation.