P-stable Nordsieck General Linear Methods for second order Ordinary Differential Equations

Raffaele D'Ambrosio, Beatrice Paternoster

Dipartimento di Matematica
via ponte don Melillo, Fisciano (Sa)
Italy
e-mail: {rdambrosio, beapat}@unisa.it

We focus our attention on the family of General Linear Methods (GLMs) in Nordsieck form, for the numerical solution of second order ordinary differential equations (ODEs). GLMs for second order ODEs are multivalue methods introduced in [2] with the aim to provide an unifying approach for the analysis of the properties of accuracy and stability, by extending the results obtained in the literature for GLMs solving first order ODEs [1, 3]. Due to their multivalue nature, GLMs transfer a certain amount r of informations from a step to the following one, which not only involve the numerical approximation of the solution in the grid points: in the special case of Nordsieck GLMs, the output vector of the method contains approximations to the solution and its scaled derivatives up to $r-1$. Special attention will be given to the derivation of highly stable GLMs, whose stability properties depend on the stability polynomial of indirect Runge-Kutta-Nyström methods based on Gauss-Legendre collocation points, which are known to be P-stable. In this way, we are able to provide P-stable GLMs whose order of convergence is greater than that of the corresponding RKN method, without heightening the computational cost. We finally provide and discuss examples of P-stable irreducible GLMs satisfying the mentioned features.

References

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