Geometric desingularization in slow-fast systems with application to the glycolytic oscillations model

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Singularly perturbed ordinary differential equations are often used to described processes which evolve on widely different time scales. Processes of this kind are very common in biochemistry, biology, neuroscience and engineering, yet their mathematical and numerical analysis are still regarded as challenging mathematical problems.

Classically, the method of matched asymptotic expansions has been used to analyze singular perturbation problems. Recently, a more qualitative approach based on methods from dynamical systems theory has developed and become known as geometric singular perturbation theory.

In this talk some concepts from this theory and geometric desingularization based on the blow-up method in the context of a specific problem describing glycolytic oscillations are presented.

Problem setting: we analyze a singularly perturbed planar system modeling oscillatory patterns in glycolysis, so-called glycolytic oscillations. In suitably scaled variables the governing equations for the variables a and b depend singularly on two small parameters ε and δ.

In the previous work by Segel and Goldbeter [3] it was argued that the system exhibits a stable limit cycle of relaxation type for $\varepsilon \ll \delta < 1$. To explain the occurrence of relaxation oscillations Segel and Goldbeter used the scaling approach. They pointed out that the disadvantage of the scaling approach is that it lies on an overall intuitive understanding of the underlying phenomenon. They suggested that it is necessary to find more systematic approaches, which are based on the mathematical structure of the equations.

Our geometric singular perturbation approach to the limit $(\varepsilon, \delta) \to (0, 0)$ completes the study presented in [3] and provides a rigorous analysis of relaxation oscillations. Particularly, the blow-up method pioneered by Dumortier and Roussarie [1] has proven to be a powerful tool in the analysis of singularly perturbed problems with non-hyperbolic points. It is shown that the blow-up method provides a clear geometric description of this fairly complicated two parameter multi-scale problem.

References

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