Energy and quadratic invariants preserving integrators of Gaussian type*

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Abstract. Recently, a whole class of energy-preserving integrators has been derived for the numerical solution of Hamiltonian problems [1,2]. In the mainstream of this research, we have defined a new family of symplectic integrators depending on a real parameter α [3]. For $\alpha = 0$, the corresponding method in the family becomes the classical Gauss collocation formula of order $2s$, where s denotes the number of the internal stages. For any given nonnull α, the corresponding method remains symplectic and has order $2s-2$: hence it may be interpreted as a $O(h^{2s-2})$ (symplectic) perturbation of the Gauss method. Under suitable assumptions, we show that the parameter α may be properly tuned, at each step of the integration procedure, so as to guarantee conservation of both energy and all quadratic invariants. The resulting energy and quadratic invariants-preserving method shares the same order $2s$ as the generating Gauss formula.

1. [1] L. Brugnano, F. Iavernaro, D. Trigiante. Analisys of Hamiltonian Boundary Value Methods (HBVMs): a class of energy-preserving Runge- Kutta methods for the numerical solution of polynomial Hamiltonian systems. (2009) *submitted* (`arXiv:0909.5659`).
2. [2] L. Brugnano, F. Iavernaro, D. Trigiante, *The Hamiltonian BVMs (HBVMs) Homepage.* (`arXiv:1002.2757`).
3. [3] L. Brugnano, F. Iavernaro, D. Trigiante. On the existence of energy-preserving symplectic integrators based upon Gauss collocation formulae. (2010) *submitted* (`arXiv:1005.1930`).

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