Error estimation techniques based on defect computation and global reconstruction

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Loosely following historical lines, we first give an overview on error estimation techniques for discretization methods based on the defect correction principle. For a given numerical approximation, its defect w.r.t. the analytical problem is computed, and a pointwise approximation for the global error is constructed via backsolving using a simple auxiliary scheme. This class of techniques can be motivated in an abstract setting, but the algorithmic details have to be properly chosen. The major requirements for such an estimator are that it should be asymptotically correct, efficient and robust, in particular on general meshes, such that it lends itself to mesh adaptation.

We show how these requirements can be satisfied by a proper reformulation of the analytical problem in combination with a related choice of the auxiliary scheme and, where required, interpolating the underlying approximation before computing the defect. The case of explicit first or second order ODE systems, discretized by finite-difference or collocation schemes, is now quite well understood. An outline of the proof of pointwise asymptotic correctness is presented, highlighting the interplay of algorithmic components which makes the procedure work. Some numerical examples are provided.

These considerations show that the design of this class of estimators is rather ODE-specific. In particular, they can be successfully applied to boundary value problems. In the second part we try to extend our point of view to other problem classes, e.g., implicit problems, elliptic PDEs, or evolution equations. This is mainly work in progress; we will propose and discuss routes for tackling some of these problems;.