On the stability and convergence behaviour of exponential operator splitting spectral methods for a class of nonlinear Schrödinger equations

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In this talk, the issue of efficient numerical methods for the space and time discretisation of nonlinear Schrödinger equations is addressed. As a model problem, the time-dependent d-dimensional Gross–Pitaevskii equation

$$\mathrm{i}\hbar\,\partial_t\Psi(x,t) = \Big(-\tfrac{\hbar^2}{2m}\Delta + V(x) + g\,\big|\Psi(x,t)\big|^2\Big)\Psi(x,t)\,, \qquad x\in\mathbb{R}^d, \quad t\geq 0,$$

arising in quantum physics for the description of Bose–Einstein condensates is considered. The main objective is to study the quantitative and qualitative behaviour of high-accuracy discretisations that rely on pseudo-spectral and exponential operator time-splitting methods. In particular, this includes a stability and convergence analysis of high-order exponential operator splitting methods for a class of nonlinear evolutionary Schrödinger equations. For this purpose, a general analytical framework and the formal calculus of Lie-derivatives is utilised. Numerical examples illustrate the theoretical results.