

Structure of B-series for some classes of geometric integrators

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Abstract

The characterizations of B-series of symplectic and energy preserving integrators are well-known. The graded Lie algebra of B-series of modified vector fields include the Hamiltonian and energy preserving classes as Lie subalgebras, these spaces are relatively well understood. However, two other important classes are the integrators which are conjugate to Hamiltonian and energy preserving methods respectively. The modified vector fields of such methods do not form linear subspaces and the notion of a grading must be reconsidered. We suggest to study these spaces as filtrations, and viewing each element of the filtration as a trivial vector bundle whose typical fiber replaces the graded homogeneous components. In particular, we shall study properties of these fibers, a particular result is that, in the energy preserving case, the fiber of degree n is a direct sum of the n th graded component of the Hamiltonian and energy preserving space. We also give formulas for the dimension of each fiber, thereby providing insight into the range of integrators which are conjugate to symplectic or energy preserving. Some examples will also be provided.