Recent Trends in the Numerical Solution of Differential Equations: Preface

Luigi Brugnano

Dipartimento di Matematica "U. Dini", Viale Morgagni 67/A, 50134 Firenze (Italy)
`http://www.math.unifi.it/~brugnano/`

The numerical solution of Differential Equations is a very active field of inverstigation. Its hystory mostly overlaps the scientific activity of J. C. Butcher, who is honoured in this Conference on the occasion of his $75th$ birthday (Best Wishes, John!). This is also a field containing many different trends of research. In particular, this symposium is devoted to some recent advances in specific topics of the field. The latter topics can be roughly collected as follows:

- numerical methods for eigenvalue and boundary value problems (papers 1, 6, 10);
- parallel solution of differential problems (paper 2);
- new methods for the numerical solution of stiff problems (papers 3, 5, 8);
- numerical methods for Hamiltonian problems (papers 4, 7);
- efficient numerical methods for problems deriving from the semi-discretization of PDEs (papers 9, 11).

In the following, we briefly describe the contents of the various papers, listed in alphabetical order of the respective authors.

Paper 1, by Aceto, Ghelardoni, and Magherini. The authors propose a family of Boundary Value Methods (BVMs) for the numerical solution of regular Sturm-Liouville problems. The methods in such family, which can be regarded as generalizations of the basic Numerov's method, can have arbitrarily high order.

Paper 2, by Amodio and Brugnano. Here, there is a review of a parallel in time approach for solving ODE-IVPs, previosly introduced by the authors, which has recently been extended for handling large-size problems. Moreover, the existing connections with subsequent approaches are sketched.

Paper 3, by Brugnano and Magherini. It is shown that block BVMs in the family of Generalized BDF (GBDF) can be conveniently used for obtaining implicit GLMs of arbitrarily high order. Moreover, their *blended implementation* makes them very appealing, from the point of view of the computational cost.

Paper 4, by Calvo, Laburta, Montijano, and Rández. The authors study the long-term behaviour of the error in the first integrals of Hamiltonian problems, when various numerical methods are considered. In particular, the case of periodic motions when geometric integrators, both symplectic and pseudo-symplectic, are used.

Paper 5, by Gonzáles-Pinto, Hernández-Abreu, and Montijano. Here, a new family of collocation Runge-Kutta methods, depending on a free parameter, is introduced and compared with RK counterparts in the family of Radau IIA methods. Advantages are obtained when the free parameter minimizes the leading term of the error.

Paper 6, by Hollevoet, Van Daele, and Vanden Berghe. A two-phase, adaptive, exponentially fitted Numerov's type method is here considered for the efficient solution of second order differential equations. Adaptivity is here used to annihilate the leading term of the error (see also paper 10).

Paper 7, by Iavernaro and Pace. The authors devise high-order block methods, which perfectly fit in the framework of block BVMs, for the efficient solution of Hamiltonian systems with a Hamiltonian function of polynomial type. Remarkably, the numerical value of the latter Hamiltonian turns out to be exactly conserved by such methods.

Paper 8, by Mazzia, Sestini, and Trigiante. The paper presents a review about B-Spline (BS) methods, a recently introduced class of BVMs which can also be interpreted as spline collocation methods. Such collocation splines, in turn, provide high order continuous extensions of the corresponding methods.

Paper 9, by Sommeijer and Verwer. Implicit-Explicit Runge-Kutta Chebyshev (IMEX RKC) methods, previously introduced by the authors for solving time-dependent PDE problems, are here improved. This is done by means of a *two-step* form of the methods, which provides a non-zero imaginary stability boundary.

Paper 10, by Van Daele, Hollevoet, and Vanden Berghe. The authors consider the numerical solution of special linear fourth-order BVPs, by means of five-points exponentially-fitted methods. The methods depend on a parameter which can be tuned, for the problem at hand, by means of a two-phase procedure, similar to that in paper 6.

Paper 11, by Wensch, Knoth, and Galant. The authors propose an improvement of split-explicit Runge-Kutta methods, utilizing a multirate time integration, for the numerical solution of the Euler equations of gas dynamics. This leads to an improvement of the stability barrier for the acoustic equation by a factor of two.