Multirate time integration for compressible atmospheric flow

J. Wensch*, O. Knoth**, A. Galant*

* TU Dresden, Inst. f. wiss. Rechmen, 01062 Dresden, Germany
** Inst. f. Troposphärenforschung, Permoserstrasse 15, 04318 Leipzig, Germany

Abstract

The Euler equations of gas dynamics in 1D formulated as balance equations of mass, momentum and energy admit wave solutions with three different characteristic velocities: the advective velocity u and much faster sound waves propagated with velocities $u \pm c_s$. Split-explicit RK methods used in numerical weather prediction utilize the multirate concept. These methods integrate the advection terms with a multistage Runge-Kutta method where the macro stepsize Δt is restricted by the CFL number of the advective velocity. The sound waves are treated by small time steps $\Delta\tau$ respecting the CFL-condition of the faster sound waves with a simpler method.

We generalise split-explicit Runge-Kutta methods by a variation of the starting point of the small timestep integration procedure and by the inclusion of fixed tendencies of previous stages. This leads to an improvement of the stability barrier for the acoustics equation by a factor of two. Order and stability analysis is based on the assumption of exact integration of fast subprocesses. We demonstarte that this analysis carries over to the case when a finite number of small steps is applied.