On the error term of exponentially fitted Numerov methods

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In this talk, we'll take a look at the leading error term of a family exponentially fitted Numerov methods, used for solving second order differential equations. We'll consider four methods, constructed with following parameters and fitting spaces:

$$\begin{array}{lll} P=-1, & K=5, & 1,t,t^2,t^3,t^4,t^5 \\ P=0, & K=3, & 1,t,t^2,t^3e^{\pm\omega t} \\ P=1, & K=1, & 1,t,e^{\pm\omega t},te^{\pm\omega t} \\ P=2, & K=-1, & e^{\pm\omega t},te^{\pm\omega t},t^2e^{\pm\omega t} \end{array}$$

The leading error term of methods in this family where $P \geq 0$, is dependent on the solution as well as on ω. Depending on P and the form of the solution, there may be different values for ω that annihilate the first error term.

We will show that by first integrating the differential equation with the classical (non-fitted, $P=-1$) Numerov method, one can construct that leading error term, and more importantly, approximate the different ω_i that reduce it to zero. After this first phase, a second integration can be performed, using the corresponding fitted method with coefficients evaluated in such an estimated $\hat{\omega}_i$. If a proper $\hat{\omega}_i$ is chosen, this second phase results in a solution with a higher order of accuracy compared to the first solution.

Furthermore, we give guidelines to choose between multiple possible values for $\hat{\omega}_i$, for methods with $P > 0$.