On stabilized integration for time-dependent PDEs

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Abstract

Time-dependent PDE problems of mixed type containing parabolic, hyperbolic and reaction terms typically exhibit wide ranges of temporal scales, challenging the time integration to avoid an expensive fully implicit treatment of the whole coupled semi-discrete system. Recently, we have developed so-called IMplicit-EXplicit (IMEX) Runge-Kutta-Chebyshev (RKC) methods. Such methods treat advection and diffusion terms explicitly and stiff reaction terms implicitly, which is attractive in higher space dimension. The stability region of this one-step method is stretched along the negative real axis. Near the origin its boundary comes close to the imaginary axis allowing the integration of slightly dissipative problems. However, since there is no intersection with the imaginary axis, problems having purely imaginary eigenvalues may impede very small temporal step sizes.

The special *two-step* form that we will discuss here has been designed to overcome this and to provide a non-zero imaginary stability boundary. This allows, for example, the integration of pure advection equations discretized in space with centered schemes, but also problems of a different origin like damped or viscous wave equations, coupled sound and heat flow problems, or the Maxwell equations coupled with nonlinear heat equations. An additional advantage is that it also simplifies the choice of temporal step sizes satisfying the von Neumann stability criterion by embedding a thin and very long rectangle inside the stability region. Embedding rectangles or other tractable domains with this purpose is an idea of Wesseling.