Problem 9

We now consider the following fractional Brusselator problem:

$$y_1^{(0.7)} = 1 - 4y_1 + y_1^2 y_2, \qquad y_2^{(0.7)} = 3y_1 - y_1^2 y_2, \qquad t \in [0, 200], \qquad y(0) = (1.2,\ 2.8)^\top,$$

For this problem, the solution approaches a limit cycle, so that it is of periodic type, as is shown in the two plots in Figure 17.

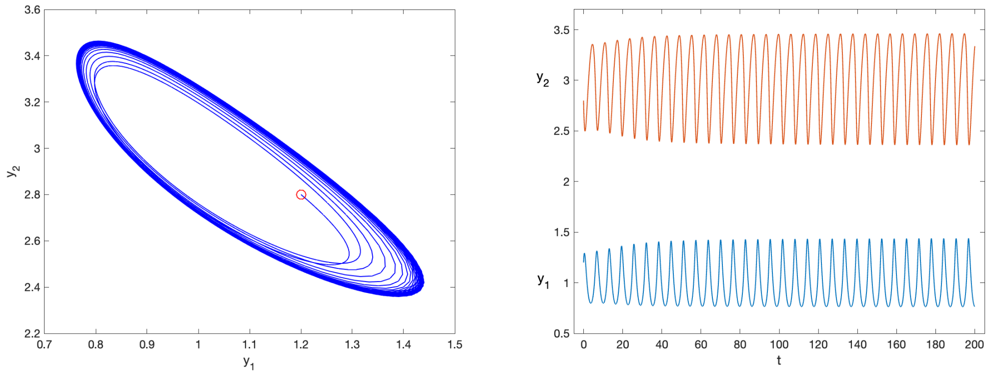

Figure 17. Problem 9: solution in the phase space (left-plot) and plotted versus time (right-plot). In the left-plot, the red circle is the starting point of the trajectory.

For this problem, since the solution is not explicitly known, a reference solution at $t = 200$ has been computed through:

```
[t,y]=fhbvm2('prob9',[1.2 2.8],200,1000,50,1000);
```

and is returned by `prob9(200)`.

We have used the codes with the following parameters:

- `fde12`, `fde12-10`: `h` $= 10^{-i}$, $i = 2, 3, 4, 5$;
- `flmm2-1`, `flmm2-2`, `flmm2-3`: `h` $= 10^{-i}$, $i = 1, 2, 3, 4$;
- `fhbvm`: `M` $= 50i$, $i = 4, 5, 6$;
- `fhbvm2`: `nu` $= 50$, `n` $= 1$, `N` $= 50i$, $i = 4, 5, 6$.

From the obtained results, depicted in Figure 18, one has that:

- `flmm2`, whichever is the method used, has a comparable performance, and is able to obtain an accuracy of at most 8 significant digits in about 400 sec;
- `fde12-10` performs better than `fde12`, but it is able to achieve less than 8 digit of accuracy (further decreasing the timestep does not improve accuracy but only increase the execution time) in about 40 sec;
- `fhbvm` and `fhbvm2` have a uniform accuracy of 13 digits, reached in about 2.5 sec and 0.5 sec, respectively.

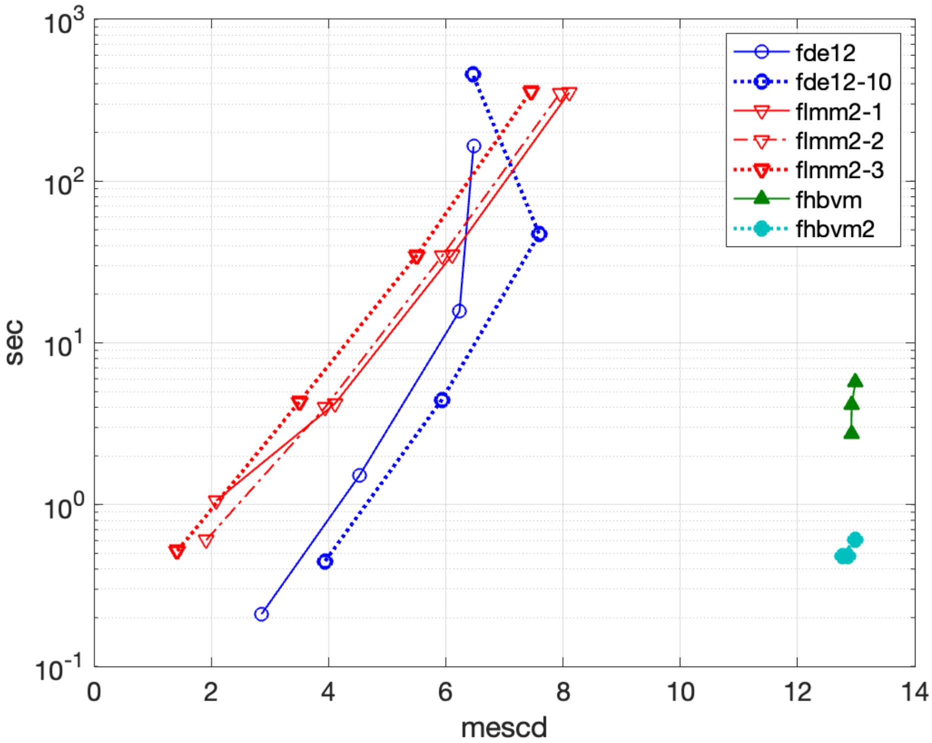

Figure 18. WPD for Problem 9.