Problem 8

This problem,

$$y_1^{(\alpha)} = \frac{\Gamma(4+\alpha)}{6}t^3 - t^{8+2\alpha} + y_2^2, \qquad y_2^{(\alpha)} = \frac{\Gamma(5+\alpha)}{24}t^4 + t^{3+\alpha} - y_1, \qquad t \in [0,2],$$
$$y_1(0) = y_1'(0) = y_2(0) = y_2'(0) = 0,$$

has solution $y_1(t) = t^{3+\alpha}$, $y_2(t) = t^{4+\alpha}$. We consider the value $\alpha = 1.25$. In such a case, both the solution (depicted in Figure 15) and the vector field are smooth at the origin.

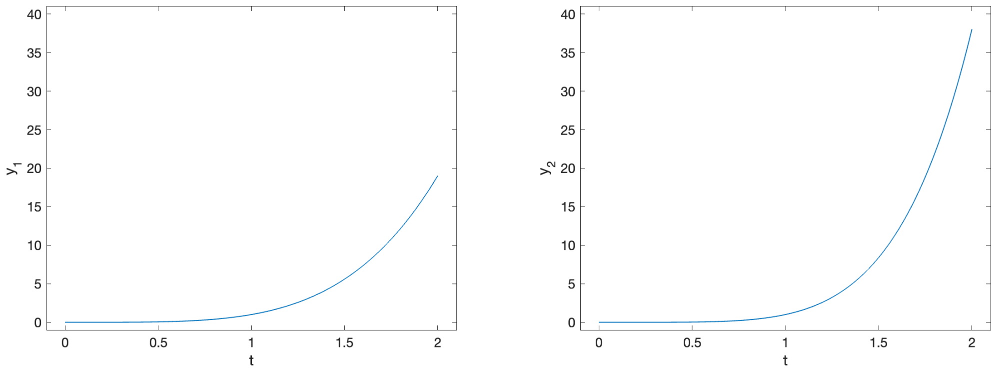

Figure 15. Components of the solution of Problem 8.

For this problem, `fcoll` and `tsfcoll` cannot be used, because the problem is a system of two FDEs. The remaining methods are used with the following parameters:

- `fde12`, `fde12-10`: $\mathtt{h} = 10^{-i}$, $i = 1,\ldots,6$;
- `flmm2-1`, `flmm2-2`, `flmm2-3` : $\mathtt{h} = 5 \cdot 10^{-i}$, $i = 1,\ldots,5$;
- `fhbvm`: $\mathtt{M} = 5,\ldots,10$;
- `fhbvm2`: $\mathtt{nu} = \mathtt{n} = 1$, $\mathtt{N} = 5,\ldots,10$.

The obtained results are reported in Figure 16, from which one can conclude that:

- `flmm2` is the less efficient code (all the 3 methods are roughly equivalent), able to reach at most 10 significant digits in 10 sec;
- `fde12` is slightly better, and more efficient than `fde12-10`, being able to achieve 11 digits of accuracy in about 6 sec;
- `fhbvm` and `fhbvm2` have a uniform accuracy of more than 15 digits with an execution time of the order of 10^{-2} sec.

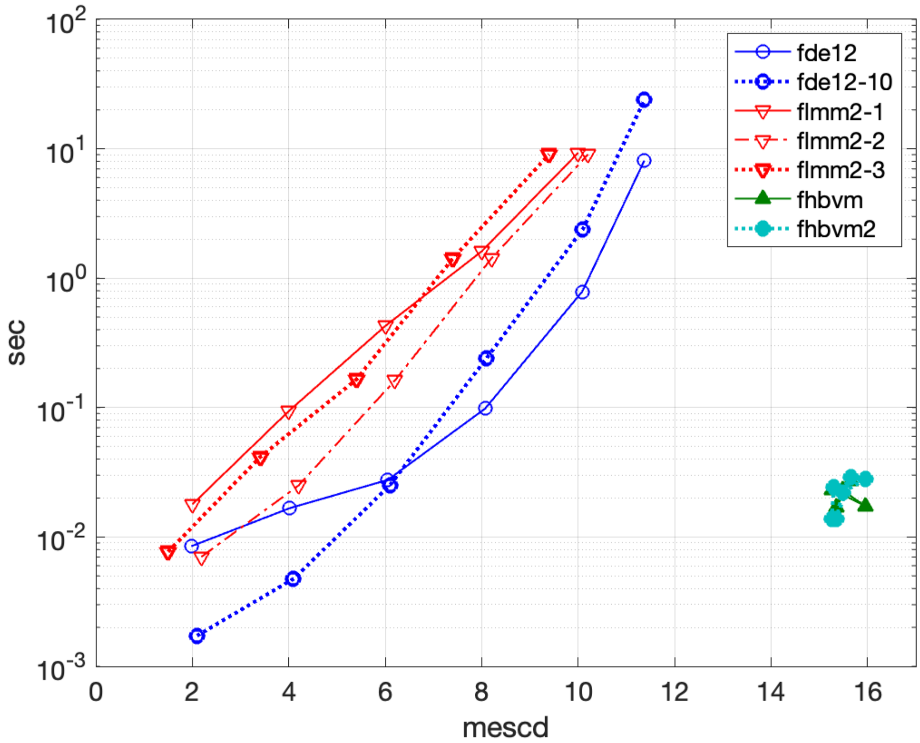

Figure 16. WPD for Problem 8.