

3.2.1. Problem 7

This is a scalar problem with both the solution and the vector field (see Figure 13) nonsmooth at the origin:

$$y^{(0.2)} = \left((1-t^2)^2 + (4+2t^{0.1}-3t^{0.3})t^{0.2} \right)^2 - y^2 + \frac{24}{\Gamma(4.8)}t^{3.8} - \frac{4}{\Gamma(2.8)}t^{1.8} - 3\frac{\Gamma(1.5)}{\Gamma(1.3)}t^{0.3} + 2\frac{\Gamma(1.3)}{\Gamma(1.1)}t^{0.1} + 4\Gamma(1.2), \quad t \in [0, 2], \quad y(0) = 1.$$

Its solution is $y(t) = (1-t^2)^2 + (4+2t^{0.1}-3t^{0.3})t^{0.2}$.

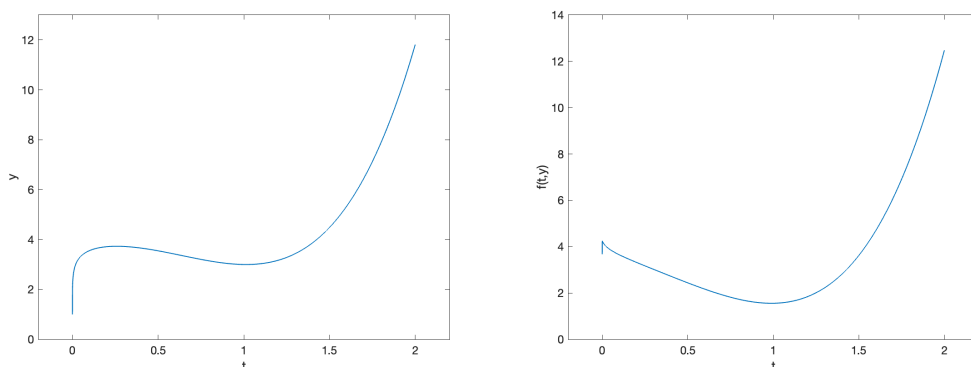


Figure 13. Problem 7: solution (left-plot) and vector field (right-plot).

We have used the parameters specified below, for the various codes:

- `fde12`, `fde12-10`, `f1mm2-1`, `f1mm2-2`, `f1mm2-3`: $h = 10^{-i}$, $i = 5, 6, 7$;
- `fcoll-3-5`, `fcoll-3-10`, `tsfcoll-3-5`, `tsfcoll-3-10`, `fcoll-4-5`, `fcoll-4-10`, `tsfcoll-4-5`, `tsfcoll-4-10`, `fcoll-5-5`, `fcoll-5-10`, `tsfcoll-5-5`, `tsfcoll-5-10`: $N = 10^i$, $i = 1, 2, 3, 4$;
- `fhbvm`: $M = 5, 6, 7, 8, 9, 10$;
- `fhbvm2`: $n = 2$, $\nu = 100$, $N = 5i$, $i = 2, 3, 4, 5, 6$.

In this case, the code `tsfcoll` does not converge, when using 5 collocation points, so that we do not report the corresponding plots (`tsfcoll` does not converge also when using 4 collocation points coupled with the coarsest values of N). The obtained results are summarized in Figure 14, from which one deduces that:

- `fde12` can reach about 3 significant digits of accuracy, with an execution time of more than 100 sec for `fde12`, and more than 500 sec for `fde12-10`;
- `f1mm2` achieves, whichever is the method used, the same accuracy of less than 6 digits, but requires about 1 hour of execution time, and about 100 sec for getting 5 digits of accuracy;
- `fcoll` has a better accuracy when considering a higher number of collocation points, and when using the larger value of the parameter r . In particular, when using 5 Gauss points and $r = 10$, it achieves an accuracy of more than 13 digits, in about 200 sec;
- `tsfcoll` does not converge, as said before, for $n = 5$, with the best results obtained by using $n = 3$ and $r = 10$. In particular, about 8 significant digits in about 280 sec;
- `fhbvm` has a uniform accuracy of about 11 significant digits, in about 0.5 sec, whereas `fhbvm2` has a uniform accuracy of 14 digits, in less than 10^{-1} sec.

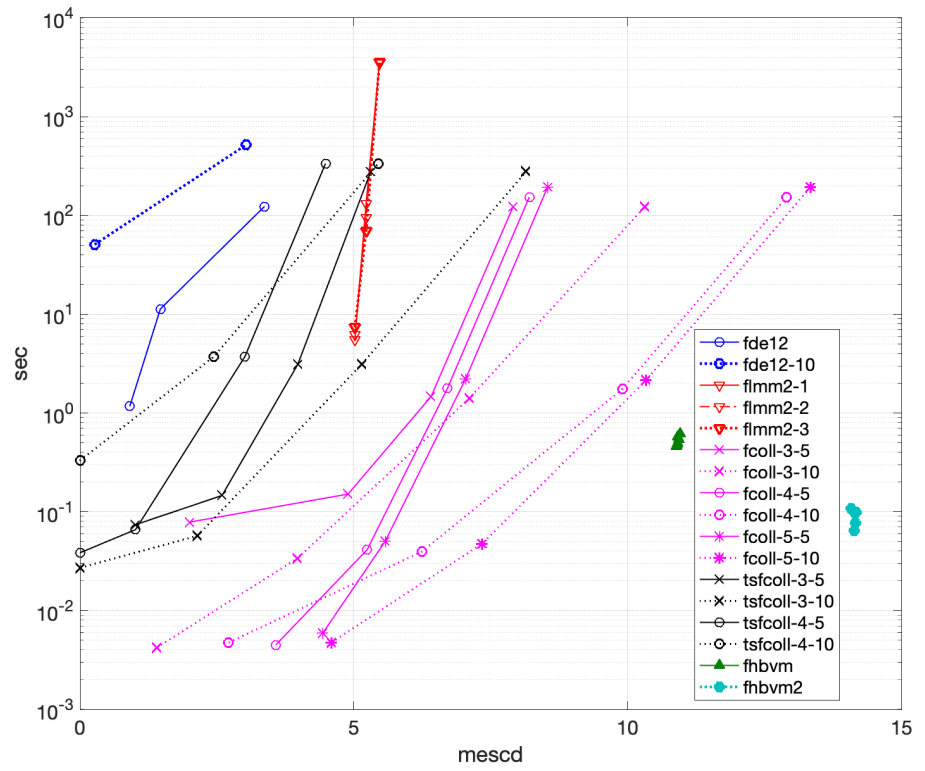


Figure 14. WPD for Problem 7.