Problem 6

This problem is given by:

$$y^{(1.5)} = \frac{y^2 - (t^{1.9}-1)^2}{2} + \frac{\Gamma(2.9)}{\Gamma(1.4)} t^{0.4}, \quad t \in [0,1], \quad y(0) = -1, \quad y'(0) = 0,$$

having solution $y(t) = t^{1.9} - 1$. For this problem, the vector field is not smooth at the origin, unlike the solution, as is shown in Figure 11.

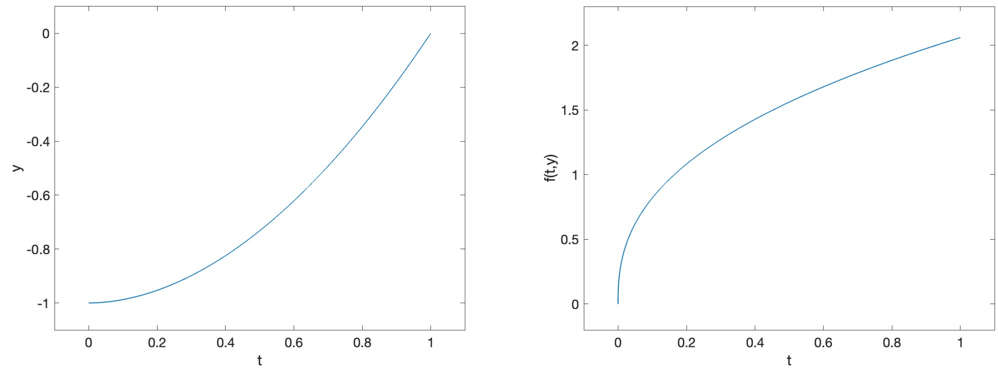

Figure 11. Problem 6: solution (left-plot) and vector field (right-plot).

We have used the parameters specified below, for the various codes:

- `fde12`, `fde12-10`, `flmm2-1`, `flmm2-2`, `flmm2-3` : $\mathtt{h} = 10^{-i}$, $i = 1,\dots,6$;
- `fcoll-3-5`, `fcoll-3-10`, `tsfcoll-3-5`, `tsfcoll-3-10`, `fcoll-4-5`, `fcoll-4-10`, `tsfcoll-4-5`, `tsfcoll-4-10`, `fcoll-5-5`, `fcoll-5-10`, `tsfcoll-5-5`, `tsfcoll-5-10`: $\mathtt{N} = 50i$, $i = 1,2,3,4$;
- `fhbvm`: $\mathtt{M} = 5,\dots,10$;
- `fhbvm2`: $\mathtt{n} = 1$, $\mathtt{nu} = 50$, $\mathtt{N} = 5i$, $i = 1,2,3,4$.

The obtained results are summarized in Figure 12, from which one deduces that:

- `fde` and `fde12-10` can reach about 9 significant digits in about 5 and 10 sec, respectively;
- `flmm2` has a similar performance as `fde12`, with the exception for the BDF2 method, which is is less accurate, though all requiring about 7 sec with the smallest timestep;
- the higher the number of the Gauss points, the more accurate is `fcoll`, which can reach about 15 significant digits in $7 \cdot 10^{-2}$ sec. The use of the smaller parameter $\mathtt{r} = 5$ turns out to be slightly more effective than the choice $\mathtt{r} = 10$;
- `tsfscoll` with 4 or 5 collocation points and $\mathtt{r} = 10$ is comparable to the best results of `fscoll`, though requiring a slightly higher execution time;
- `fhbvm` reaches a uniform accuracy of about 11 significant digits with a very low execution time (less than $2 \cdot 10^{-2}$ sec);
- `fhbvm2` has a uniform accuracy of about 15 significant digits with a very low execution time (about $1.5 \cdot 10^{-2}$ sec).

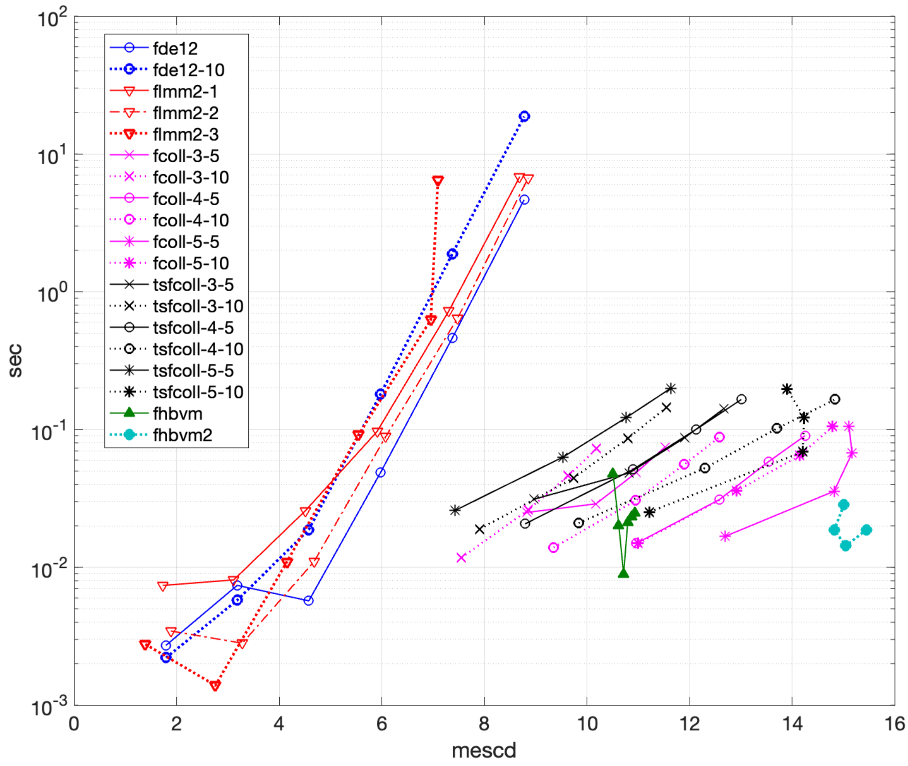

Figure 12. WPD for Problem 6.