

Problem 5

This is a nonlinear problem:

$$y^{(\alpha)} = -y^{3/2} + \frac{8!}{\Gamma(9-\alpha)}t^{8-\alpha} - 3\frac{\Gamma(5+\alpha/2)}{\Gamma(5-\alpha/2)}t^{4-\alpha/2} + \left(\frac{3}{2}t^{\alpha/2} - t^4\right)^3 + \frac{9}{4}\Gamma(\alpha+1),$$

$$t \in [0, 1], \quad y(0) = 0,$$

having solution

$$y(t) = t^8 - 3t^{4+\alpha/2} + \frac{9}{4}t^\alpha.$$

We consider the value $\alpha = 0.3$. It is worth noticing that, even though the solution is not smooth at 0, as is shown in the left-plot of Figure 9, nevertheless, the vector field is very smooth, as it can be deduced from the right-plot in the same figure.

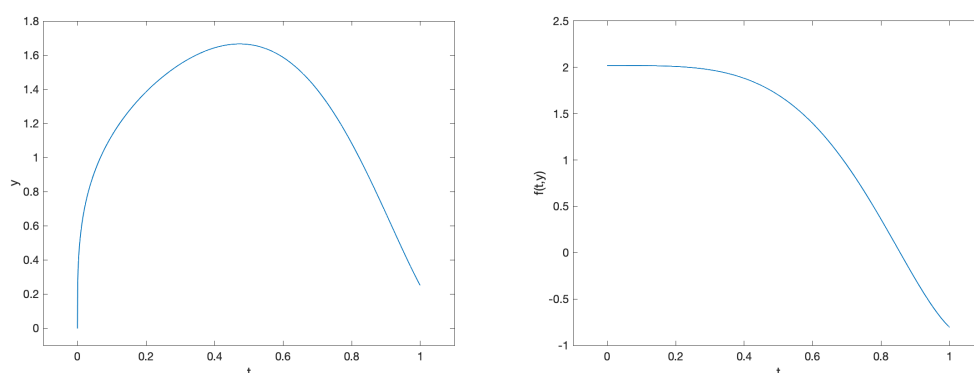


Figure 9. Problem 5: solution (left-plot) and vector field (right-plot).

This problem has a smooth vector field everywhere, so that we have used all codes with a uniform mesh, with the only exception of `tsfcoll`, which performs quite badly with a uniform mesh: for this reason, we used this code with $r = 10$. Further, we have used the parameters specified below:

- `fde12, fde12-10, flmm2-1, flmm2-2, flmm2-3`: $h = 10^{-i}$, $i = 1, \dots, 5$;
- `fcoll-3-1, fcoll-4-1, fcoll-5-1, tsfcoll-3-10, tsfcoll-4-10, tsfcoll-5-10`: $N = 25i$, $i = 2, \dots, 6$;
- `fhbvm`: $M = 5i$, $i = 1, 2, 3, 4$;
- `fhbvm2`: $nu = n = 1$, $N = 5i$, $i = 1, 2, 3, 4$.

The obtained results are summarized in Figure 10, from which one deduces that:

- `fde12-10` can reach 10 significant digits in 2 sec, thus performing better than `fde12`;
- `flmm2-1` reaches about 11 mescd, and is slightly more accurate than `flmm2-2`, in turn slightly more accurate than `flmm2-3`, all methods requiring about 1 sec;
- the higher the number of the Gauss points, the more accurate is `fcoll`, which can reach about 14 significant digits in about $2 \cdot 10^{-1}$ sec;
- `tsfcoll` is generally much less effective than `fcoll`, and when using 5 collocation points it has a very low accuracy;
- `fhbvm` and `fhbvm2` have a uniform accuracy of about 15 significant digits with a very low execution time ($2 \cdot 10^{-2}$ and $6 \cdot 10^{-3}$ sec, respectively).

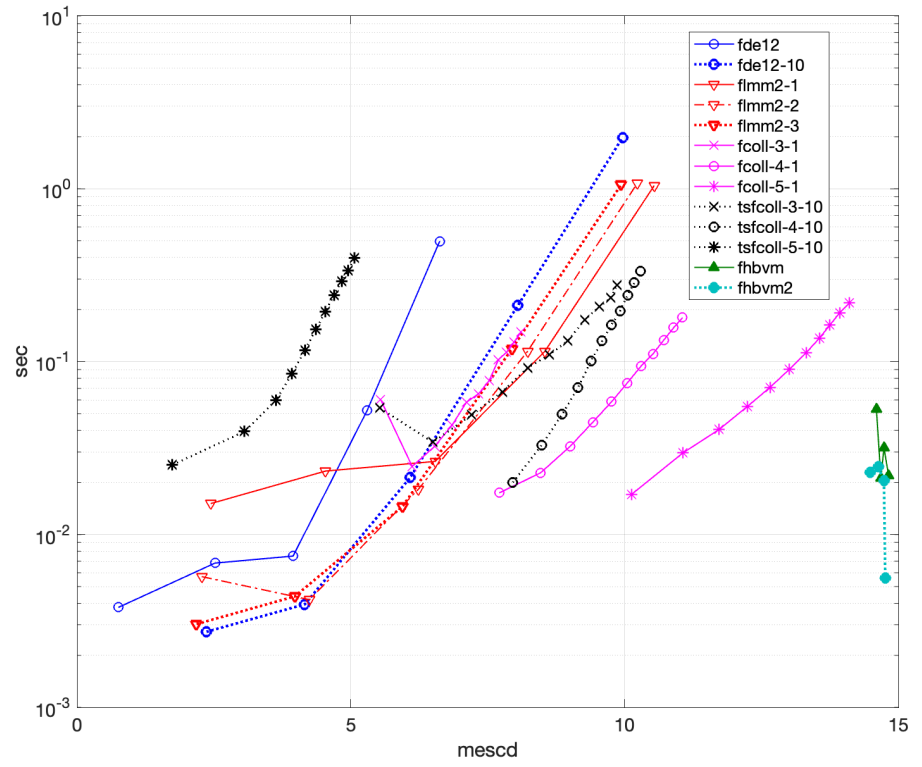


Figure 10. WPD for Problem 5.