Problem 3

This problem is a *stiff* problem, since it has two decaying modes, a slow one and a fast one:

$$y^{(0.5)} = \frac{1}{5}\begin{pmatrix} -92 & -87 \\ -58 & -63 \end{pmatrix} y - \frac{1}{10}\begin{pmatrix} 67 \\ 83 \end{pmatrix}, \qquad t \in [0, 10^2], \qquad y(0) = \begin{pmatrix} 5 \\ 10 \end{pmatrix}.$$

Its solution is depicted in Figure 5. As before, we have chosen the width of the integration interval in order to grasp the convergence to the limit point $\bar{y} = (2, -2.5)^\top$.

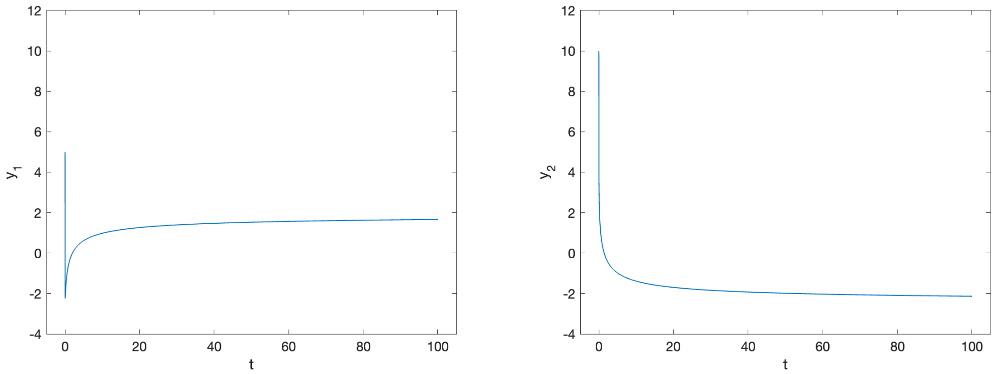

Figure 5. Components of the solution of Problem 3.

For this problem we cannot consider the codes `fcoll` and `tsfcoll`, which are only able to solve scalar problems. We use, therefore, the following codes, with the parameters fixed as follows:

- `fde12`, `fde12-10`: $\mathtt{h} = 10^{-i}$, $i = 4, 5, 6$;
- `flmm2-1`, `flmm2-2`, `flmm2-3` : $\mathtt{h} = 10^{-i}$, $i = 2, 3, 4, 5$;
- `fhbvm`: $\mathtt{M} = 5, \ldots, 10$;
- `fhbvm2`: $\mathtt{nu} = 50$, $\mathtt{n} = 1$, $\mathtt{N} = 10i$, $i = 5, \ldots, 10$.

The obtained results are summarized in Figure 6, from which one deduces that:

- `fde12` is able to reach about 3.5 significant digits in slightly more than 700 sec (`fde12-10` does not improve accuracy but only increases the execution time);
- `flmm2` is able to reach 5 significant digits in about 10^3 sec, when using both the trapezoidal rule or the Newton-Gregory formula, whereas the version using BDF2 is less accurate, though requiring comparable execution times;
- `fhbvm` and `fhbvm2` are the most efficient codes, reaching about 13 and 14 digits of accuracy, respectively, in less than $2 \cdot 10^{-1}$ sec.

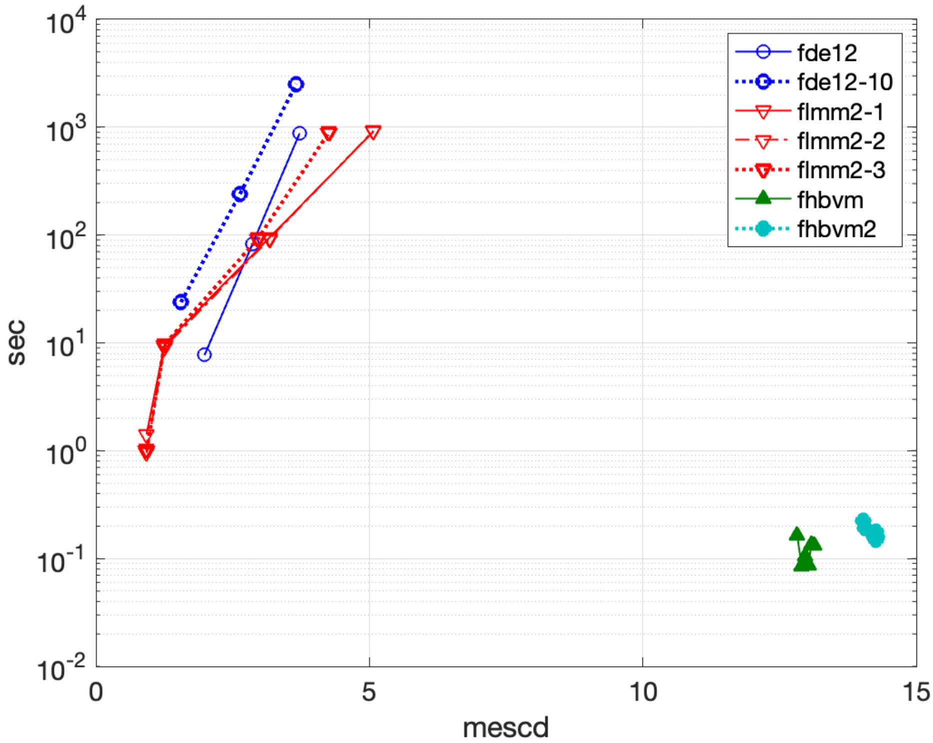

Figure 6. WPD for Problem 3.