Problem 2

This problem is scalar, with a mildly fast decaying mode:

$$y^{(0.5)} = -20y + 20, \qquad t \in [0, 10], \qquad y(0) = 10.$$

In this case, $y(t) \to 1$, as $t \to \infty$. The solution is depicted in Figure 3. We have chosen the width of the integration interval in order to observe the convergence to the limit point.

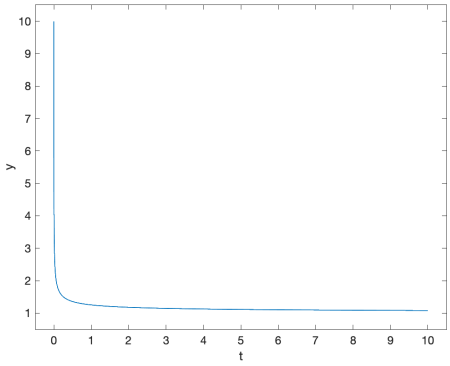

Figure 3. Solution of Problem 2.

For building the WPD for this problem, we used the following parameters for the codes:

- `fde12`, `fde12-10`: $\texttt{h} = 10^{-i}$, $i = 4, \ldots, 7$;
- `flmm2-1`, `flmm2-2`, `flmm2-3`: $\texttt{h} = 10^{-i}$, $i = 3, \ldots, 6$;
- `fcoll-3-5`, `fcoll-3-10`, `tsfcoll-3-5`, `tsfcoll-3-10`: $\texttt{N} = 500i$, $i = 1, \ldots, 5$;
- `fcoll-4-5`, `fcoll-4-10`, `tsfcoll-4-5`, `tsfcoll-4-10`: $\texttt{N} = 250i$, $i = 1, \ldots, 5$;
- `fcoll-5-5`, `fcoll-5-10`, `tsfcoll-5-5`, `tsfcoll-5-10`: $\texttt{N} = 100i$, $i = 1, \ldots, 5$;
- `fhbvm`: $\texttt{M} = 5, \ldots, 10$;
- `fhbvm2`: $\texttt{N} = \texttt{n} = 1$, $\texttt{nu} = 25i$, $i = 2, \ldots, 6$;

In this case, `tsfcoll`, when using 5 collocation points, does not converge, so that we do not report the corresponding results, as well as those obtained with 4 collocation nodes, for which the same problem occurs in most of the runs.

The obtained results are depicted in Figure 4:

- the two codes `fhbvm` and `fhbvm2` turn out to be the most effective ones, among those considered, with a uniformly high accurate solution (about 14 mescd) and a negligible execution time (less than $5 \cdot 10^{-2}$ sec);
- the highest order collocation `fcoll` implementations are the most effective, and appear to be not so much sensitive to the choice of the parameter `r` used for the graded mesh (almost 12 digits of accuracy are obtained in about 0.5 sec);
- differently, the larger the parameter `r` is, the better the accuracy of the computed solution by `tsfcoll` with 3 collocation points (which is the only one always converging). In particular, `tsfcoll-3-10` can reach an accuracy comparable to that of `fhbvm` and `fhbvm2`, thought with a much higher execution time (6 sec vs. $4 \cdot 10^{-2}$ and 10^{-2} sec, respectively).

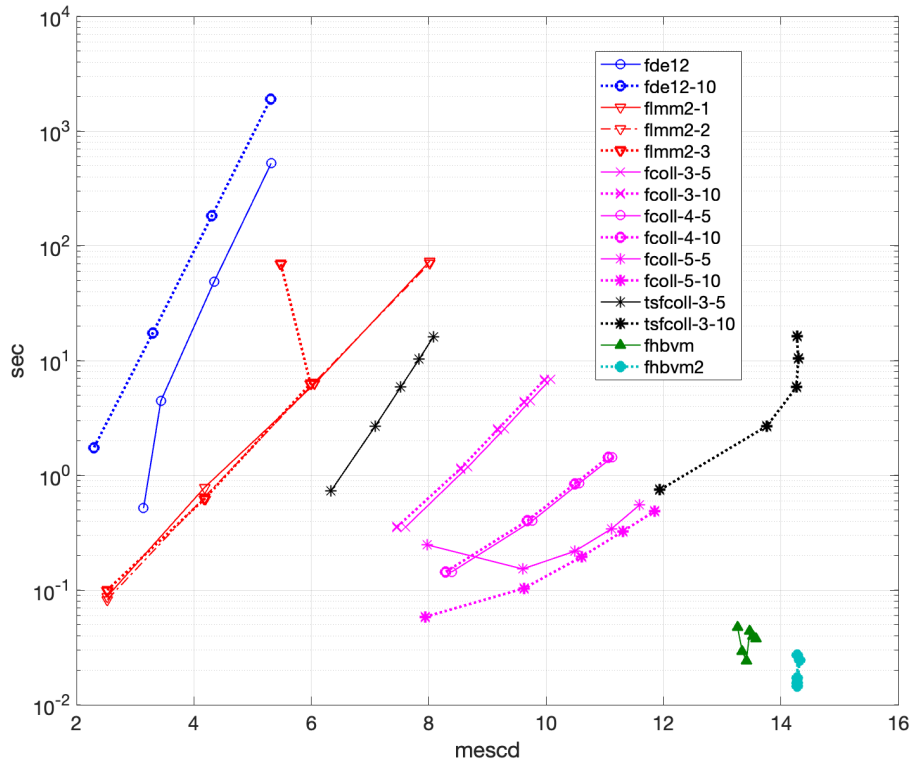

Figure 4. WPD for Problem 2.