Problem 10

This is a fractional version of the Van der Pol problem:

$$y_1^{(0.9)} = y_2, \qquad y_2^{(0.9)} = -y_1 - 10y_2(y_1^2 - 1), \qquad t \in [0, 30], \qquad y(0) = (0, -2)^\top.$$

As in the ODE case, the solution approaches a limit cycle, as is shown in the left-plot of Figure 19, so that the solution becomes eventually periodic (see also the right-plot in the same figure).

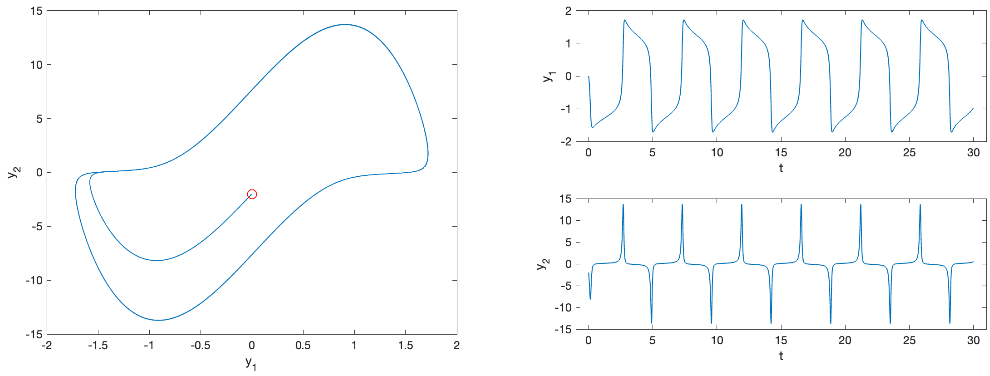

Figure 19. Problem 10: solution in the phase space (left-plot) and plotted versus time (right-plot). In the left-plot, the red circle is the starting point of the trajectory.

For this problem, since the solution is not explicitly known, a reference solution at $t = 30$ has been computed through:

```
[t,y]=fhbvm2('prob10',[0 -2],30,1000,50,1000);
```

This reference vector is returned by `prob10(30)`. Moreover:

- `fde12` and `fde12-10` do not converge even when using a timestep as small as $\texttt{h} = 10^{-6}$;
- `flmm2-1`, `flmm2-2`, and `flmm2-3` exhibit problems in the convergence of the nonlinear iteration, even when using a timestep as small as $\texttt{h} = 10^{-5}$.

Consequently, we have not consider them. The remaining two codes have been used with the following parameters:

- `fhbvm`: $\texttt{M} = 50i$, $i = 4, 5, 6, 7$;
- `fhbvm2`: $\texttt{nu} = 50$, $\texttt{n} = 1$, $\texttt{N} = 50i$, $i = 4, 5, 6, 7$.

From the obtained results, reported in Figure 20, one infers that both the codes `fhbvm` and `fhbvm2` can reach an accuracy of 12 digits in about 4 sec and 0.6 sec, respectively.

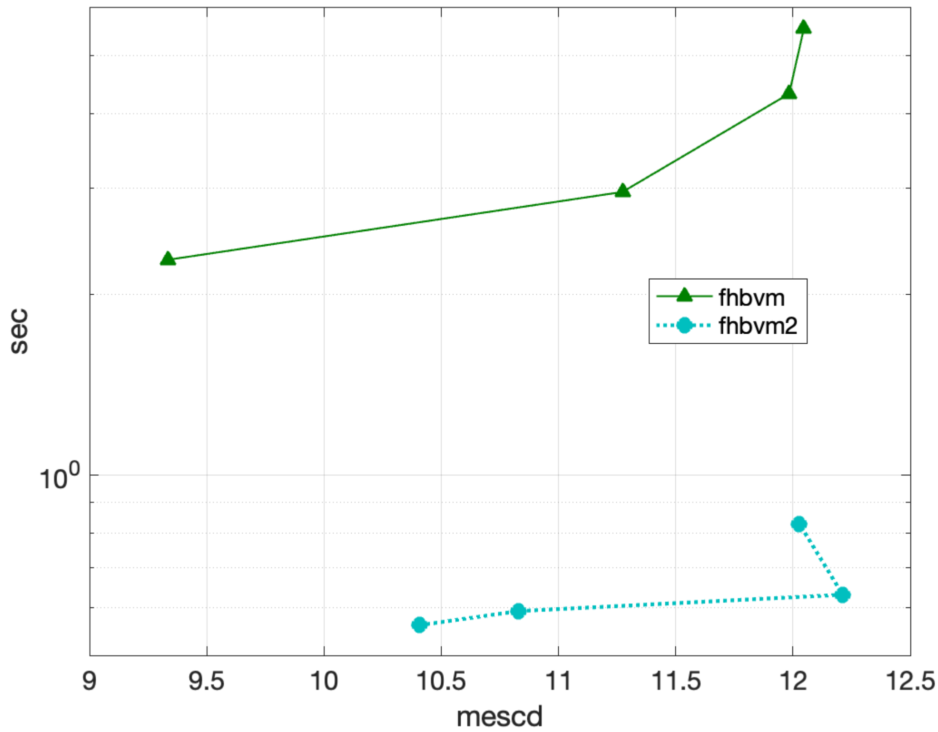

Figure 20. Problem 10.