1) $u = \frac{1}{2} 4^{1-5} = \frac{1}{2} 4^{-4} = 2^{-9}$.

2) $\frac{1}{x} \to \kappa = 2$; $\sqrt{x} \Rightarrow \kappa = \frac{1}{2}$; $x+y \to \kappa = \frac{|x|+|y|}{|x+y|}$.

3) $f(0)=0$, $f'(x) = \sin 4x + 4x\cos 4x \big|_{x=0} = 0$, $f''(x) = 4\cos 4x + \ldots \big|_{x=0} \neq 0 \Rightarrow m=2$

4) $f(x)=0 \to \quad x_{n+1} = x_n - \dfrac{f(x_n)}{\frac{f(x_n)-f(x_{n-1})}{x_n - x_{n-1}}}$, $n \geq 0$, x_0, x_1 dati.

5)
```
function x = sec(fun, x0, x1, toll, itmax)
% commenti
f0= feval(fun, x0); f1=feval(fun,x1);
err= abs(x1-x0)/(1+abs(x1));
for i=1:itmax
    den= (f1-f0)/(x1-x0);
    x= x1- f1/den;
    err= abs(x-x1)/(1+abs(x1));
    if err <= toll, break, end
    x0=x1; f0=f1;
    x1=x; f1= feval(fun, x1);
end
if i>=itmax & err>toll, warning('...'); end
return
```

6) $\kappa = \dfrac{1}{|f'(x)|}\Big|_{x=0} = \dfrac{1}{|5e^{5x}\sin x + e^{5x}\cos x|}\Big|_{x=0} = \dfrac{1}{1} = 1$.

7) $\|A\|_1 = \max\{2+|\alpha|, 1+|\beta|\}$; $\|A\|_\infty = \max\{2, 1+|\beta|, |\alpha|\}$, $\|A\|_2 \leq \sqrt{\|A\|_1 \cdot \|A\|_\infty}$.

8) $\forall (\alpha,\beta) \neq (0,-1)$. $A^TA = \begin{bmatrix} 2+\alpha^2 & 1-\beta \\ 1-\beta & 1+\beta^2 \end{bmatrix} = \begin{bmatrix} 1 & \\ \frac{1-\beta}{2+\alpha^2} & 1 \end{bmatrix} \begin{bmatrix} 2+\alpha^2 & \\ & d \end{bmatrix} \begin{bmatrix} 1 & \frac{1-\beta}{2+\alpha^2} \\ & 1 \end{bmatrix}$

con $d = 1+\beta^2 - \dfrac{(1-\beta)^2}{2+\alpha^2}$.

9) $\underline{v}_1 = \begin{pmatrix} 1+\sqrt{2+\alpha^2} \\ 1 \\ \alpha \end{pmatrix}$, $\underline{v}_2 = \begin{pmatrix} 1+\sqrt{1+\beta^2} \\ -\beta \\ 0 \end{pmatrix}$;

10) Questo lo abbiamo fatto in esercitazione.

11) $A^{-1} = \begin{bmatrix} \cos\theta & -\sin\theta & 0 & 0 \\ \sin\theta & \cos\theta & 0 & 0 \\ 0 & 0 & 1 & \\ 0 & 0 & -10\theta & 1 \end{bmatrix}$. $\Rightarrow \|A\| = \|A^{-1}\| = \max\{|\sin\theta|+|\cos\theta|, 1+10\theta\} \leq 1+10\pi$, per $\theta \in [0,\pi]$

$\Rightarrow \kappa(A) \leq (1+10\pi)^2$.

12) $x: \min \|r\|_2^2 = \|A\underline{x}-\underline{b}\|_2^2 = \|QRx - b\|_2^2 = \|Q(Rx - \underbrace{Q^Tb}_{=\underline{g}})\|_2^2 = \|Rx - \underline{g}\|_2^2$ $\quad R = \begin{pmatrix} \hat{R} \\ 0 \end{pmatrix}$

$\hat{R}$ $n \times n$, $g_1 \in \mathbb{R}^n$ $\quad = \left\| \begin{pmatrix} \hat{R} \\ 0 \end{pmatrix} x - \begin{pmatrix} g_1 \\ g_2 \end{pmatrix} \right\|_2^2 = \|\hat{R}x - g_1\|_2^2 + \|g_2\|_2^2$

$= \|g_2\|_2^2$, scegliendo x soluzione di $\hat{R}x = g_1$. La soluzione esiste, perché $\hat{R}$ è nonsingolare.