Symmetrizations of convex sets and convergence of their iterations

Gabriele Bianchi, Richard J. Gardner e Paolo Gronchi

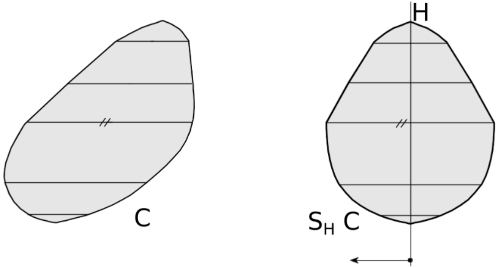
MathAnalysis(at)UniFiPiSi II, Pisa, November 2019

G. Bianchi, R.J. Gardner and P. Gronchi, *Symmetrizations in Geometry*, Adv. Math. 2017

————, *Convergence of Symmetrization Processes*, arXiv 2019

Let us begin with some examples: Steiner

Let H be an hyperplane
Steiner symmetrization with respect to H of a convex body C, denoted by $S_H\, C$:



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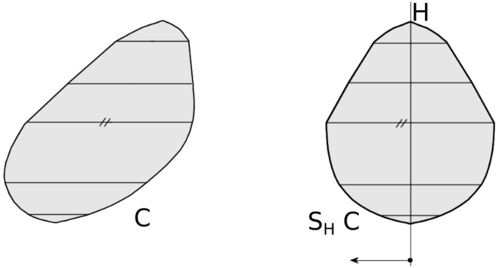
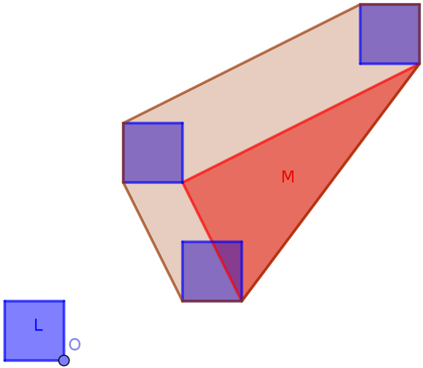


- does not change volume
- in general, it decreases surface area

Minkowski symmetrization: preliminaries

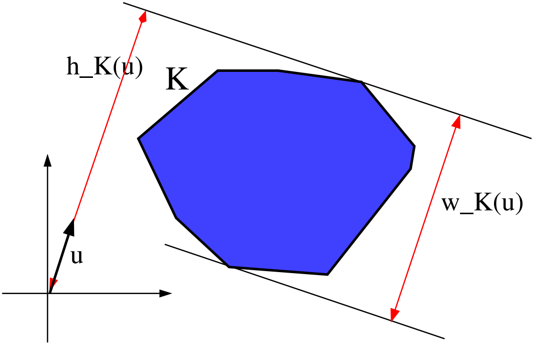
Minkowski sum of L and M

$$L + M = \{x + y \,:\, x \in L, y \in M\}$$
$$= \bigcup_{y \in M} (L + y)$$



Minkowski symmetrization: preliminaries

Support function $h_K(u)$ and width $w_K(u)$



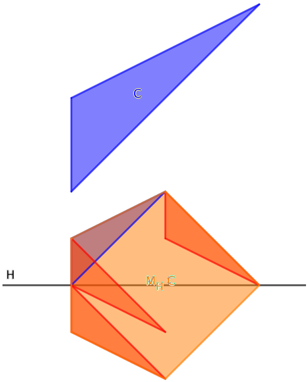
Mean width$=\int_{S^{n-1}} w_K(u)\,du$

Minkowski symmetrization

Let H be a subspace of dimension i,
$1 \leq i \leq n-1$.
Minkowski symmetry with respect to H of convex body C:

$$M_H C = \frac{1}{2} C + \frac{1}{2} R_H C$$

where R_H denotes reflection with respect to H.



- What do I mean by R_H? if $x \in \mathbb{R}^n$ and $x = h + h' \in H \times H^\perp$ then $R_H x = h - h'$.

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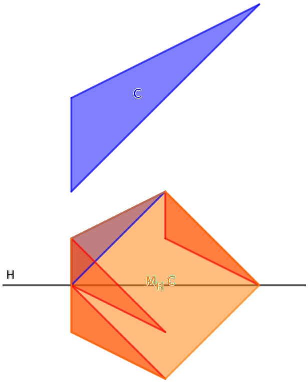

- What do I mean by R_H? if $x \in \mathbb{R}^n$ and $x = h + h' \in H \times H^{\perp}$ then $R_H x = h - h'$.
- M_H is linear: $M_H(K+L) = M_H K + M_H L$
- M_H does not change mean width
- in general, M_H increases surface area and volume

Iterating the symmetrizations in order to converge to a ball

let $\diamondsuit_H$ denote Steiner or Minkowski symmetrization
It is known that there are sequences (H_m) of hyperplanes such that, for any choice of the convex body C, as $m \to \infty$

$$\left(\diamondsuit_{H_m} \diamondsuit_{H_{m-1}} \dots \diamondsuit_{H_1} C\right) \to \text{ball}.$$

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- Ingredient of a proof of the isoperimetric inequality in the class of convex bodies

plan of the talk

In this research we have:

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- studied their most meaningful properties and the relations existing among them;
- characterized Steiner and Minkowski symmetrizations on the basis of some of these properties;
- applied these ideas to the study of the convergence to a ball of iterations of symmetrizations.

Definition of i-symmetrization

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$$\text{Any map} \qquad \diamondsuit_H : \mathcal{E} \to \mathcal{E}_H$$

where

- $\mathcal{E} = \{\text{convex bodies}\}$ or $\mathcal{E} = \{\text{compact sets}\}$,
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some of the properties which appear to be relevant:

- monotonicity (wrt inclusion): $K_1 \subset K_2 \Longrightarrow \diamondsuit K_1 \subset \diamondsuit K_2$
- $\mathcal{F}$-preserving ($\mathcal{F}$ is a functional): $\mathcal{F}(K) = \mathcal{F}(\diamondsuit K)$
- invariance on H-symmetric sets: $\diamondsuit K = K$ for every H-symmetric K
- invariance on H-symmetric cylinders
- invariance wrt translations orthogonal to H of H-symmetric sets: $\diamondsuit(K + x) = K$ for every H-symmetric set K and $x \in H^{\perp}$

An unified definition of Steiner and Minkowski symmetrization which shows their duality

an unified dual definition of Steiner e Minkowski symm.

Theorem

For every i and $K \in \{\text{convex bodies}\}$ we have

$$F_H K = \bigcup_{y \in H^\perp} (K+y) \cap R_H(K+y)$$

and

$$M_H K = \bigcap_{y \in H^\perp} conv\Big((K+y) \cup R_H(K+y)\Big)$$

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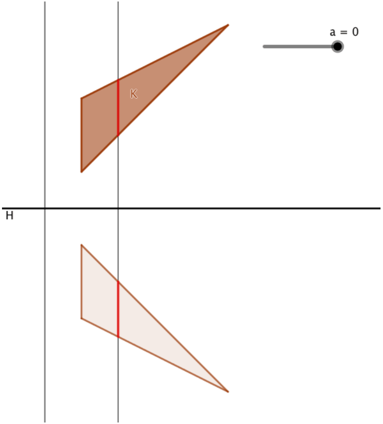
- in the next slides we visualize the theorem and give an idea of its proof for $i = n-1$



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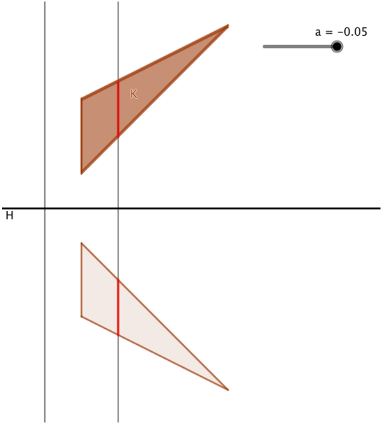

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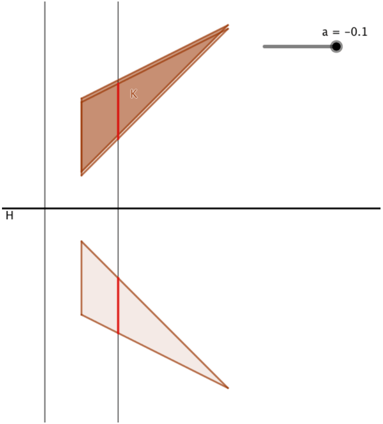

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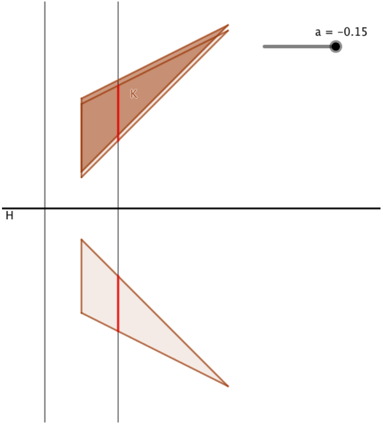

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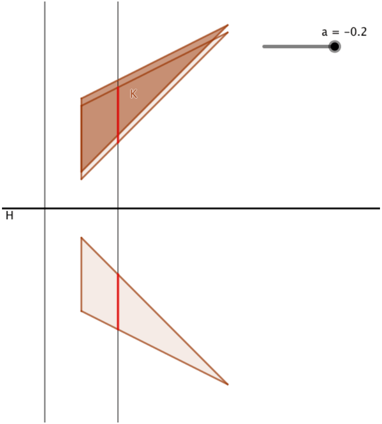

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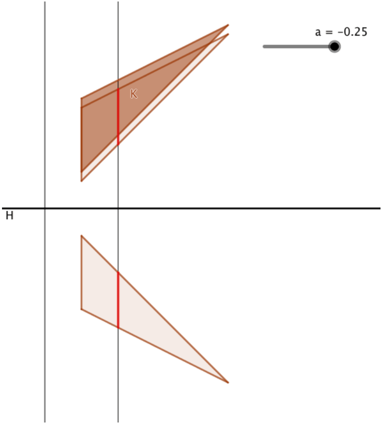

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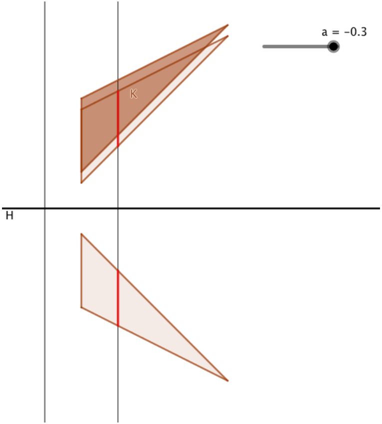

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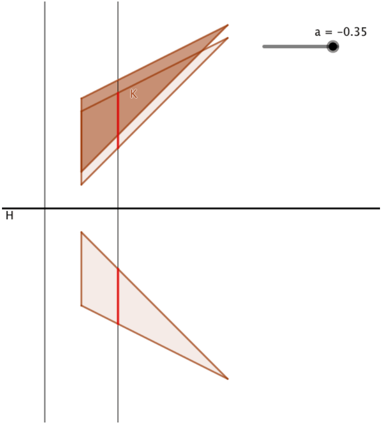

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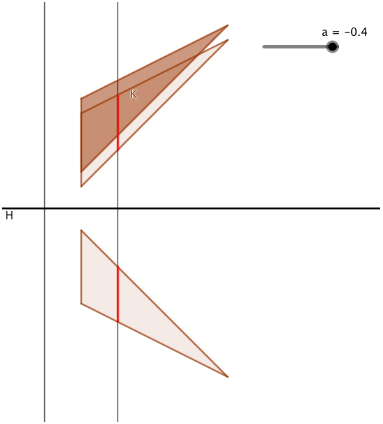

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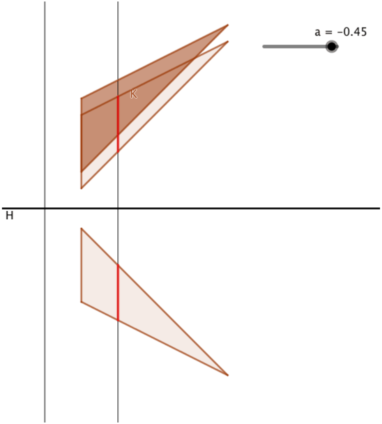

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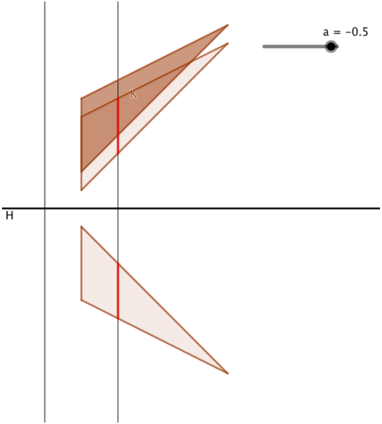

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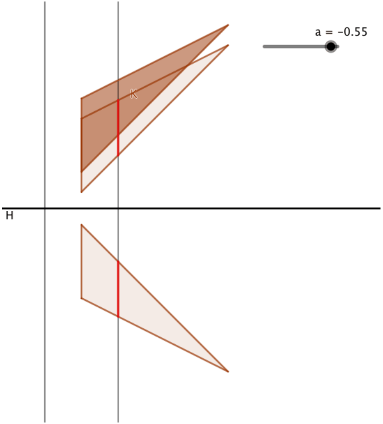

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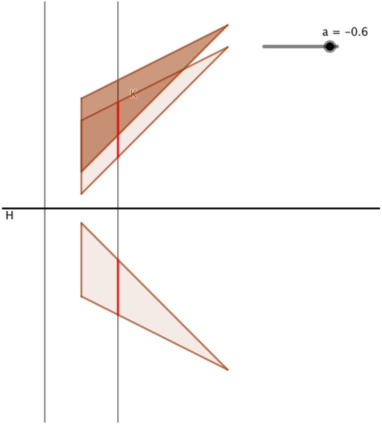

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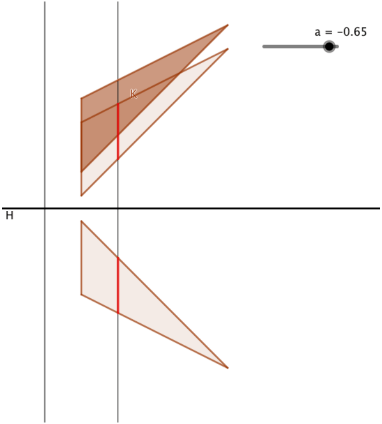

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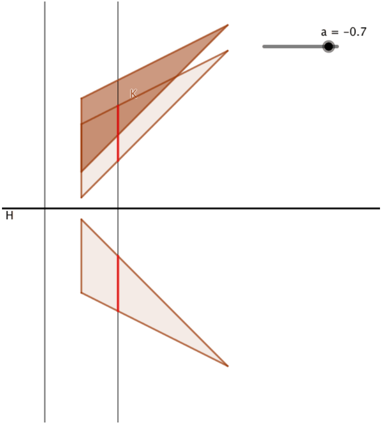

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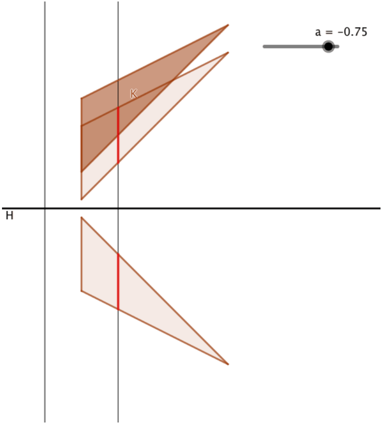

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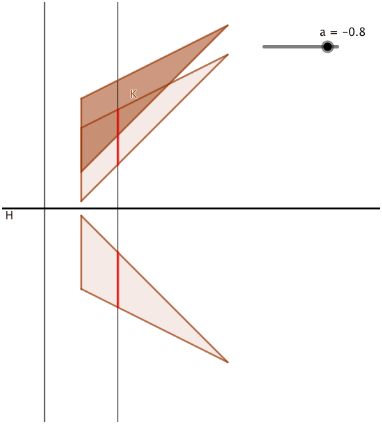

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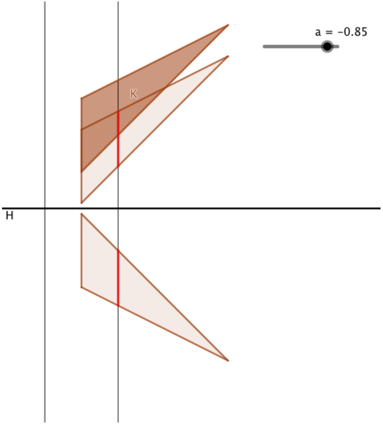

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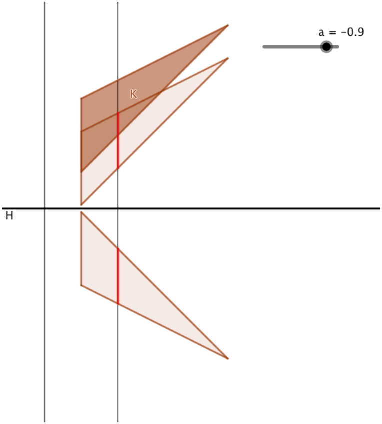

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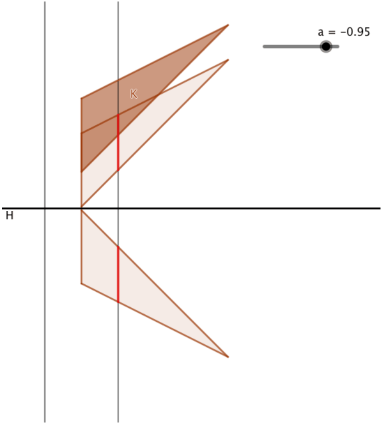

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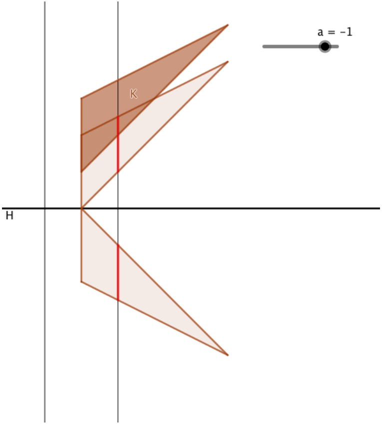

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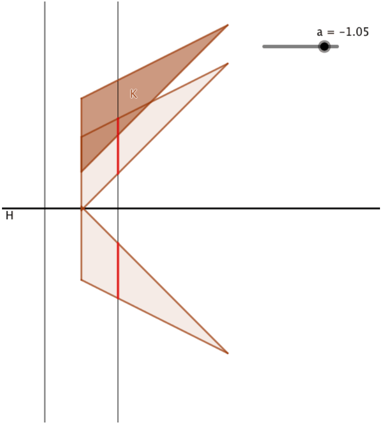

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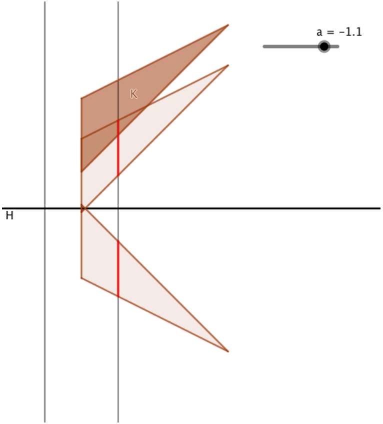

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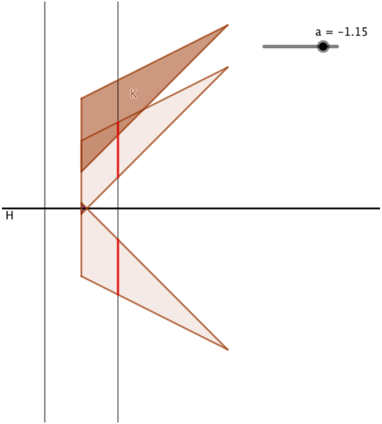

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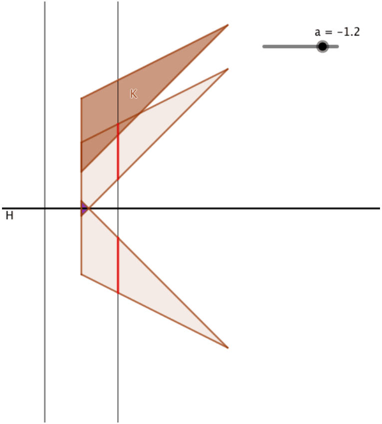

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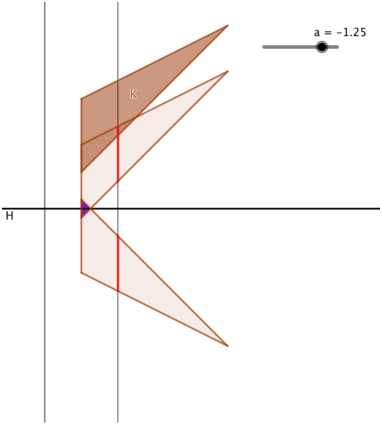

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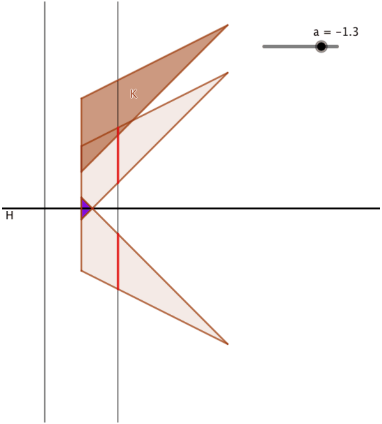

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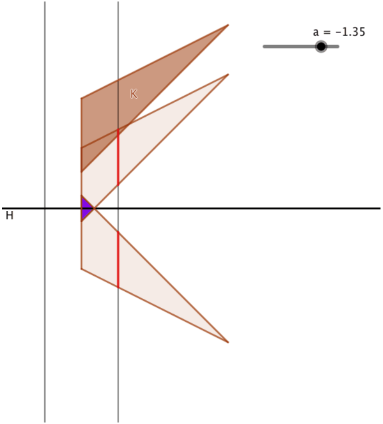

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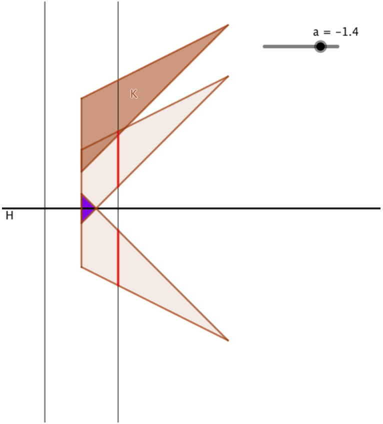

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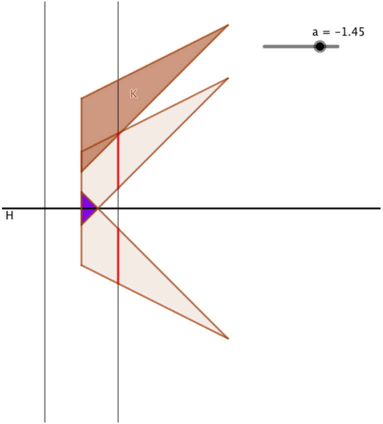

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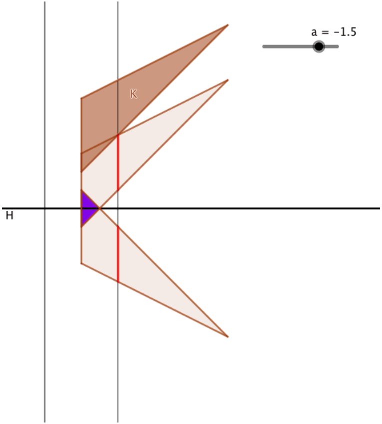

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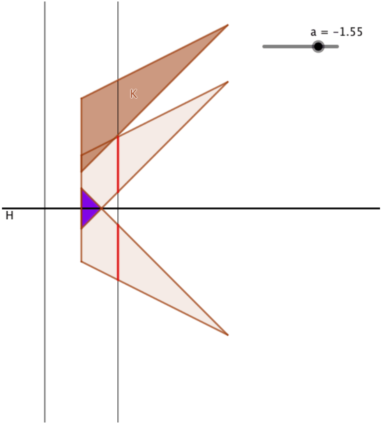

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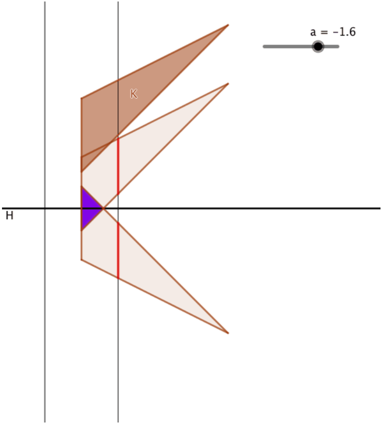

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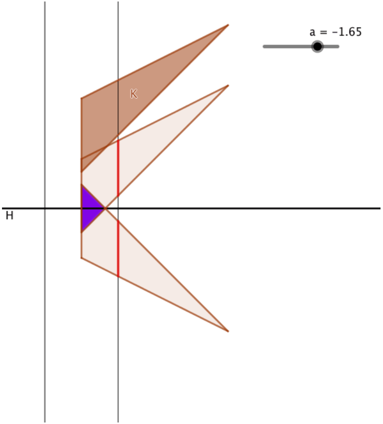

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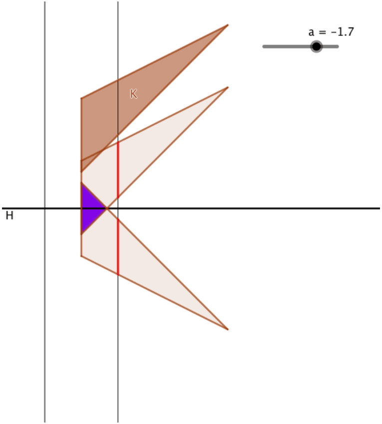

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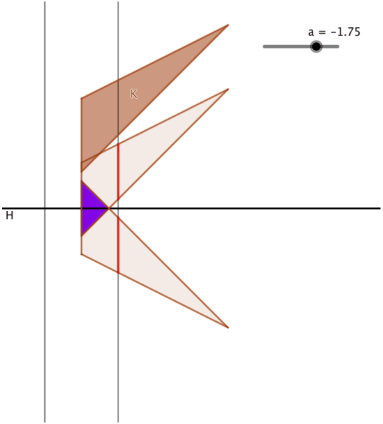

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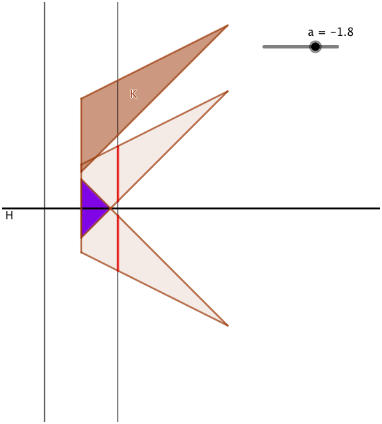

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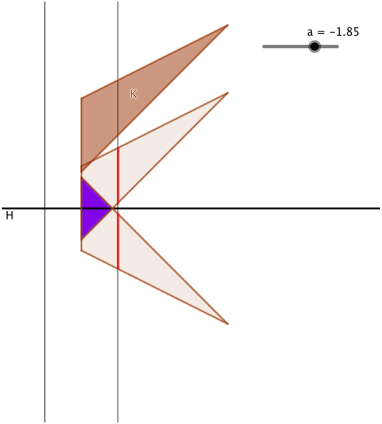

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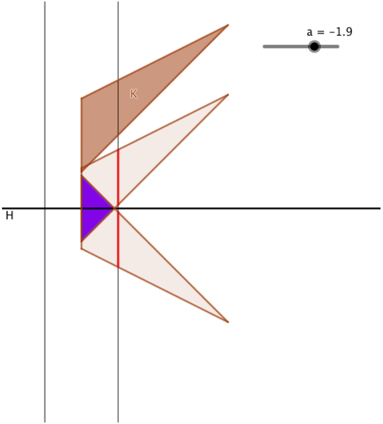

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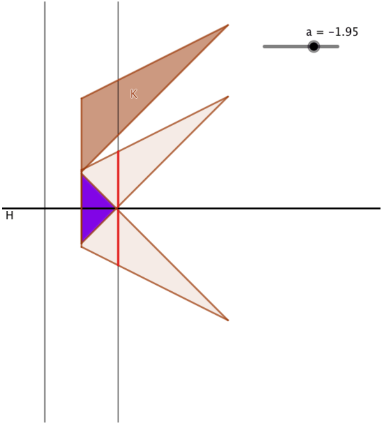

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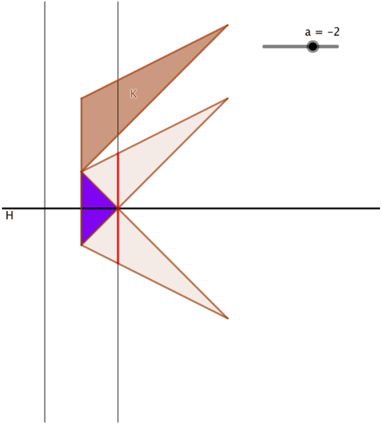

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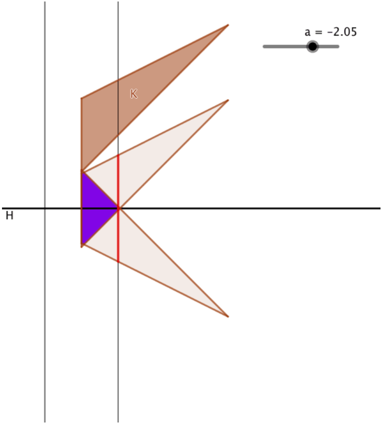

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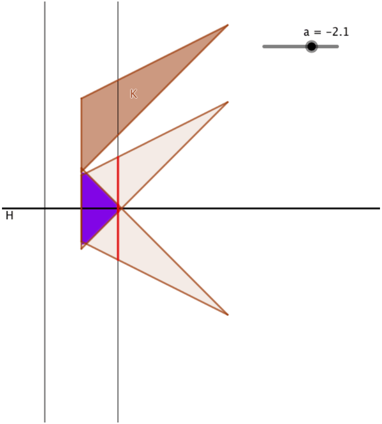

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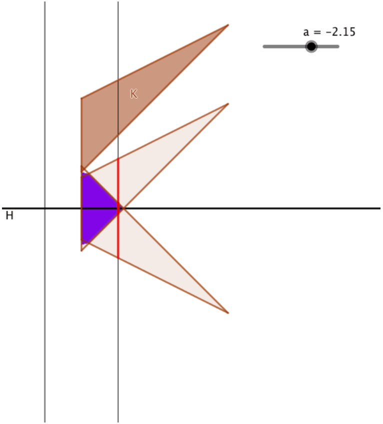

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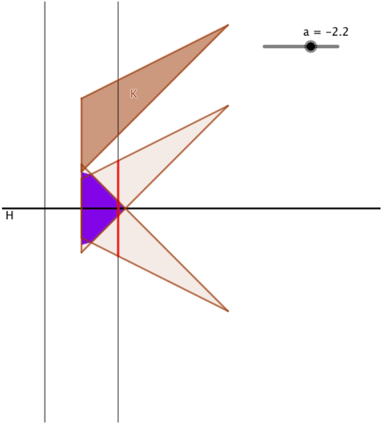

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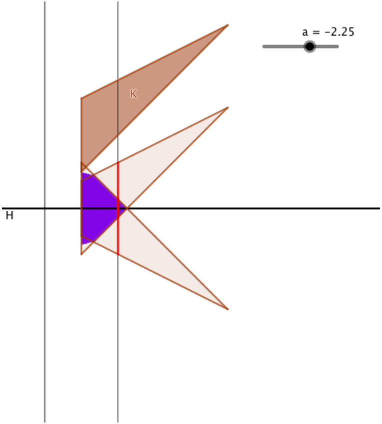

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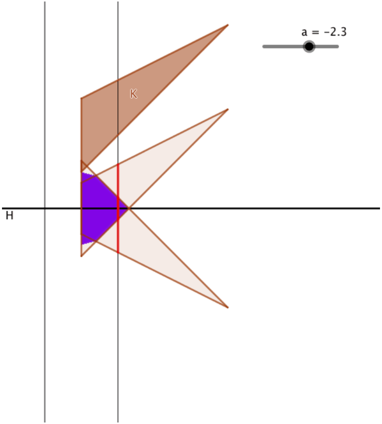

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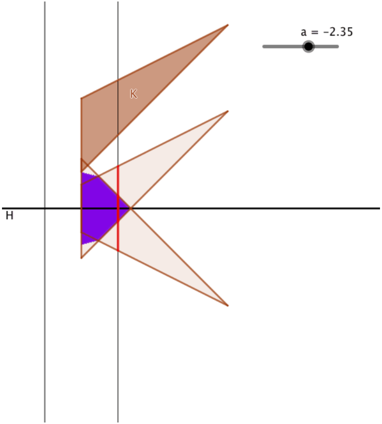

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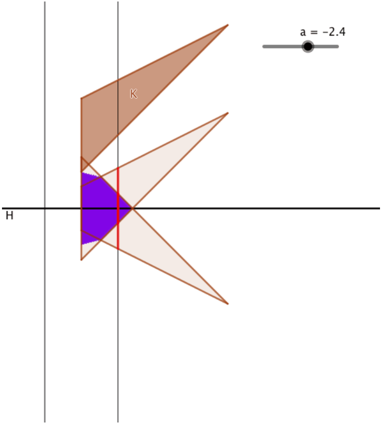

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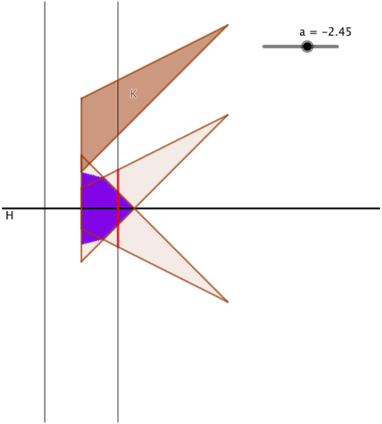

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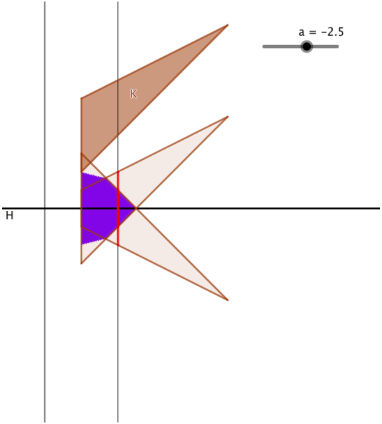

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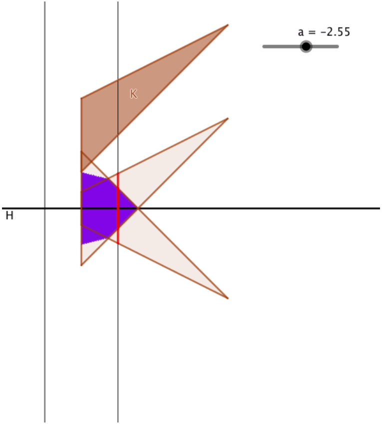

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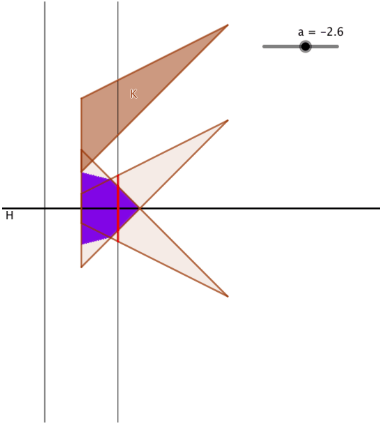

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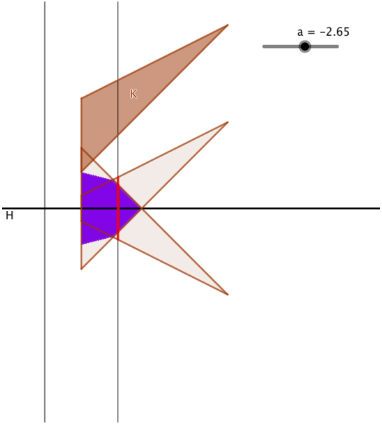

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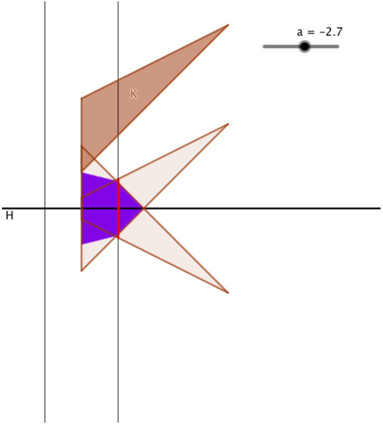

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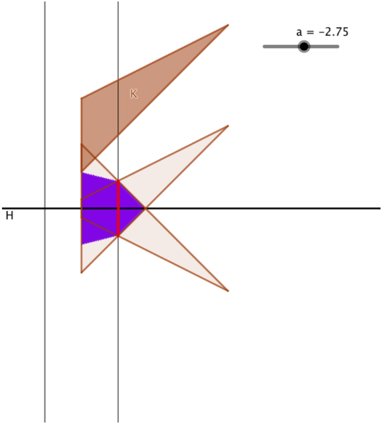

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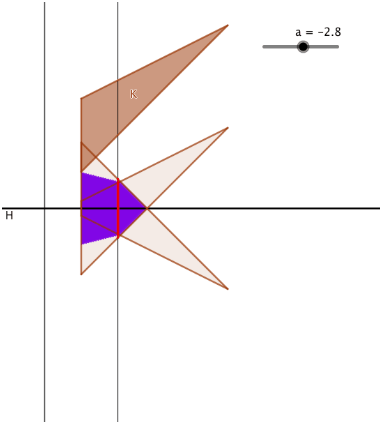

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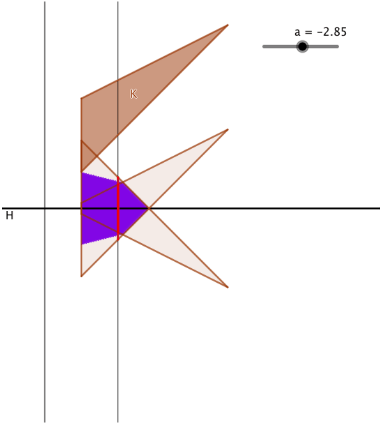

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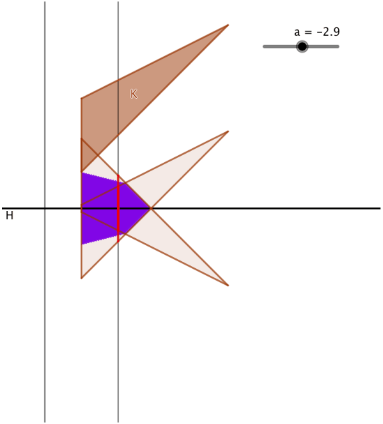

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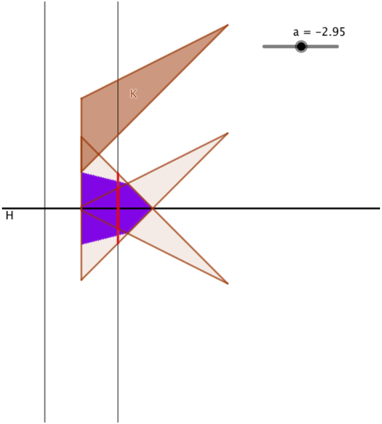

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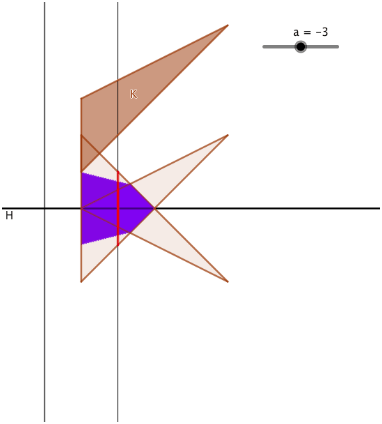

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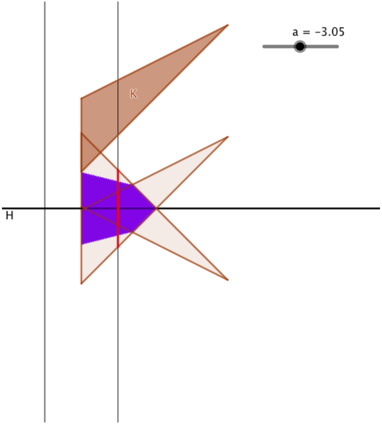

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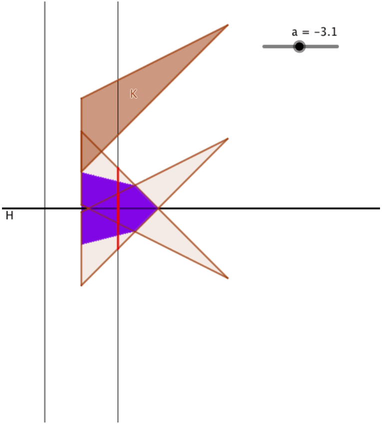

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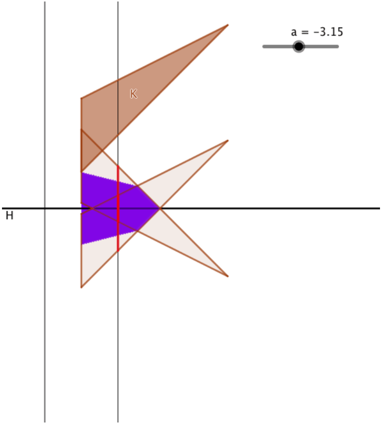

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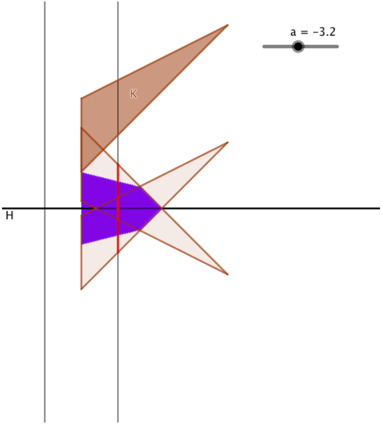

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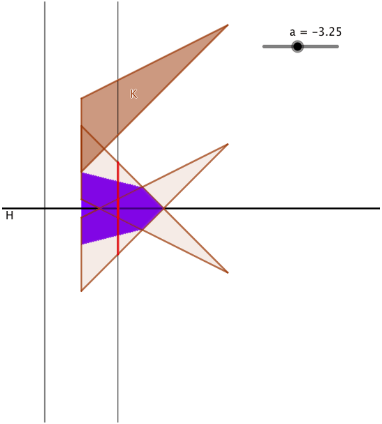

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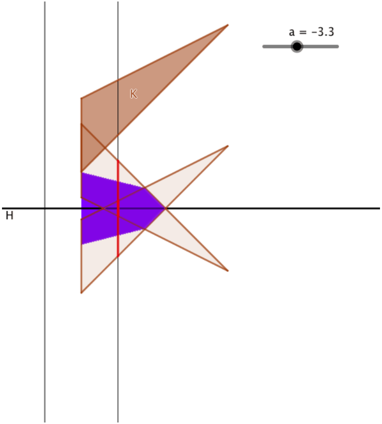

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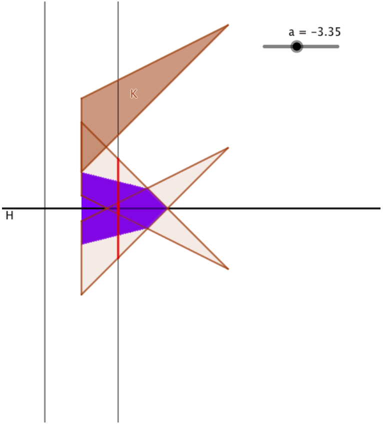

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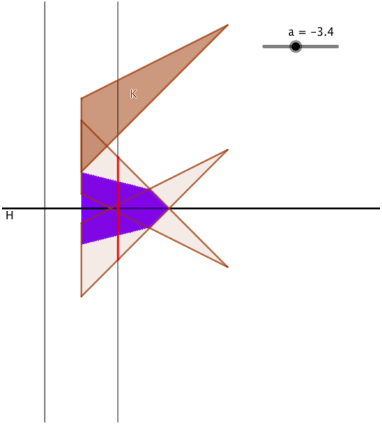

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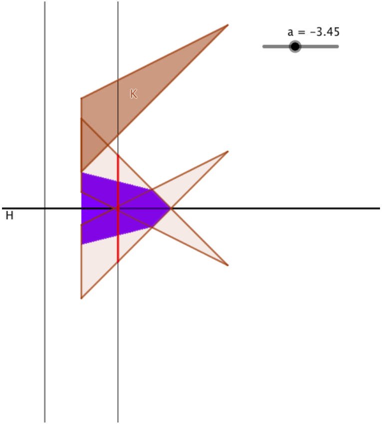

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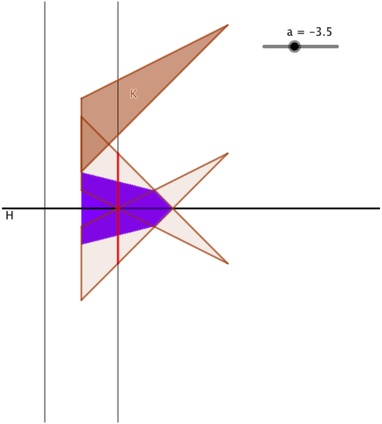

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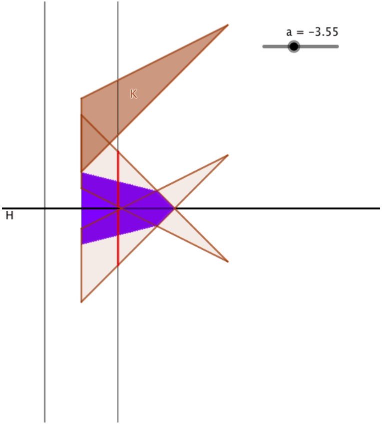

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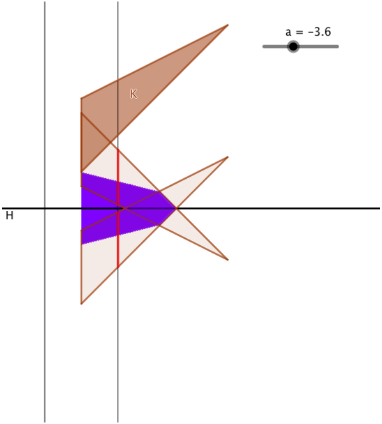

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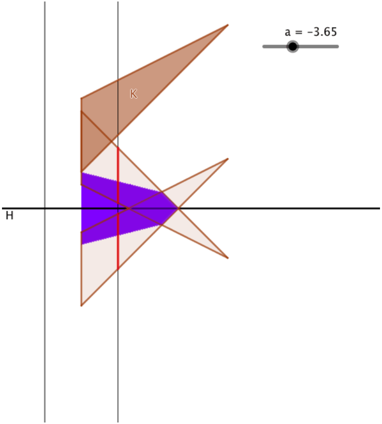

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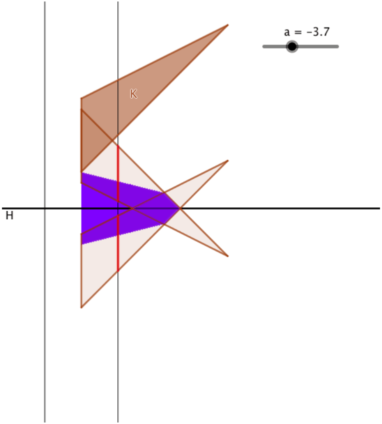

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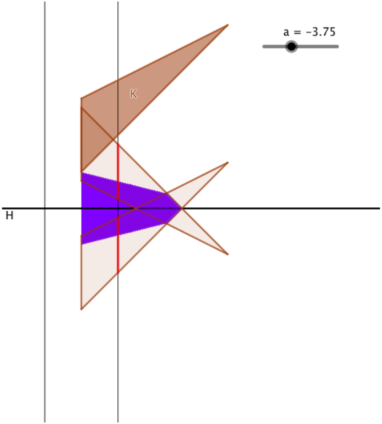

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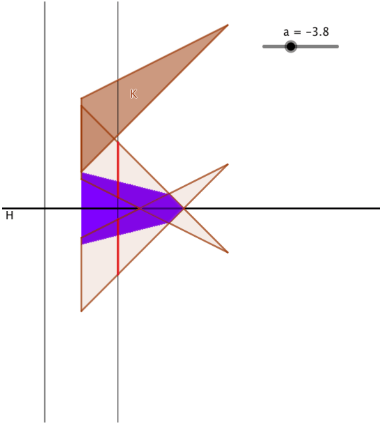

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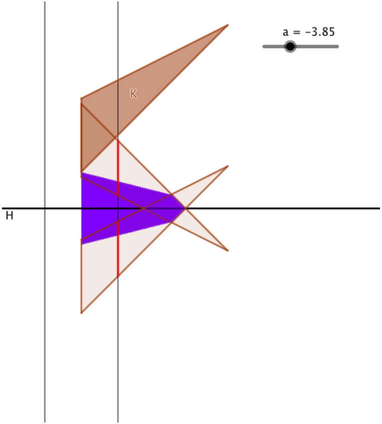

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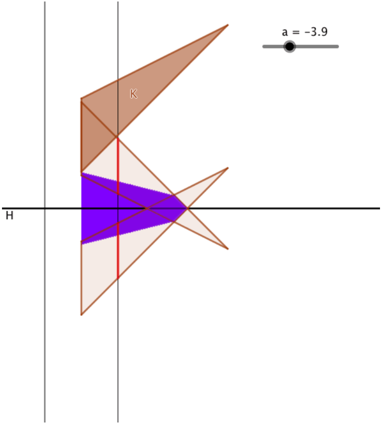

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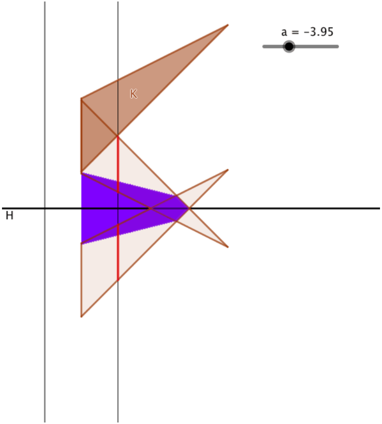

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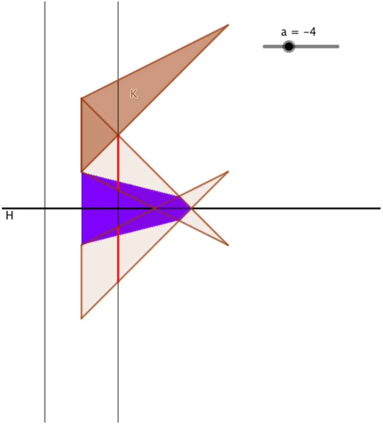

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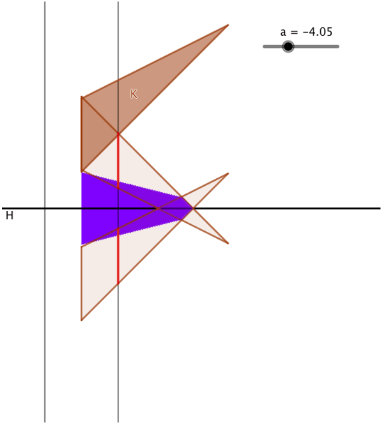

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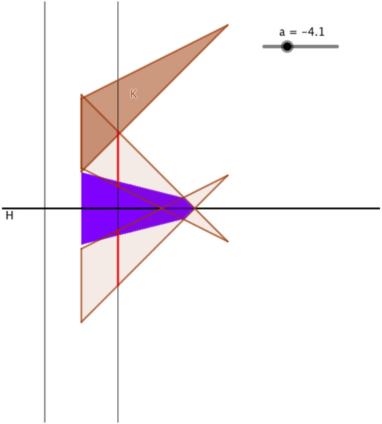

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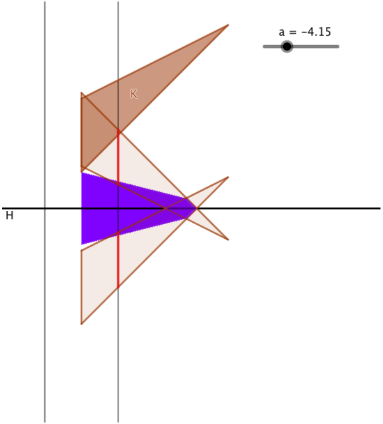

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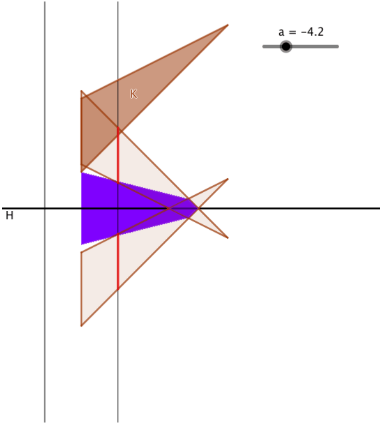

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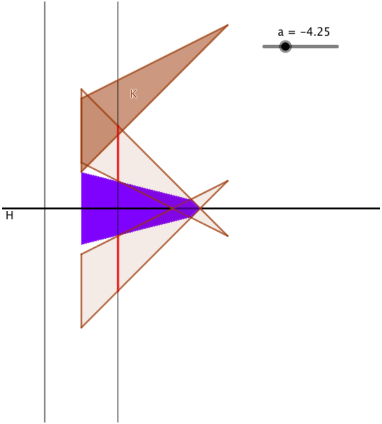

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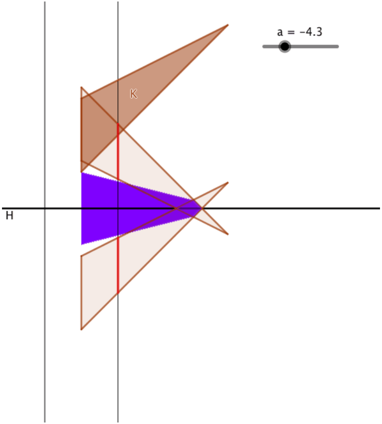

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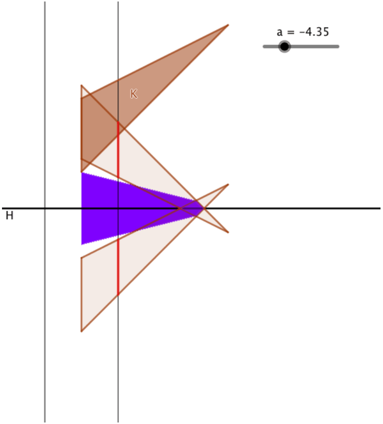

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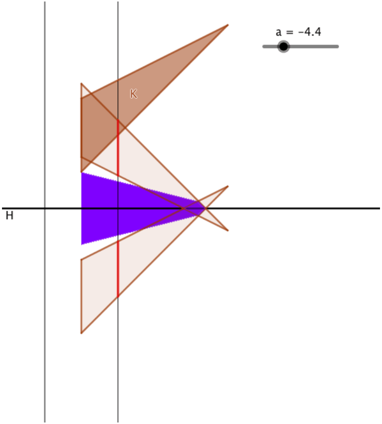

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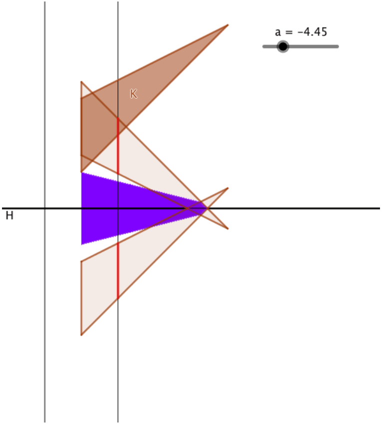

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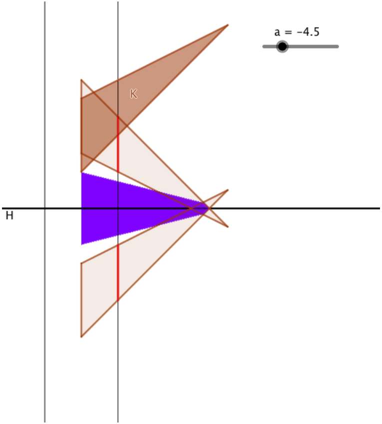

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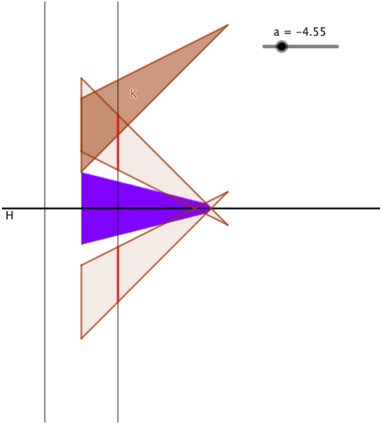

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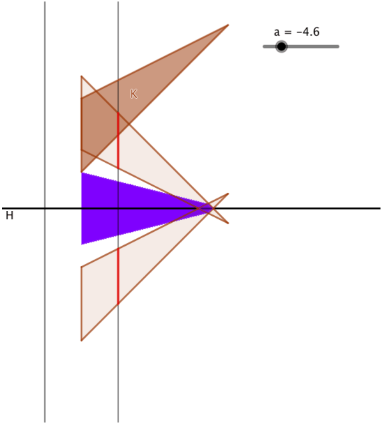

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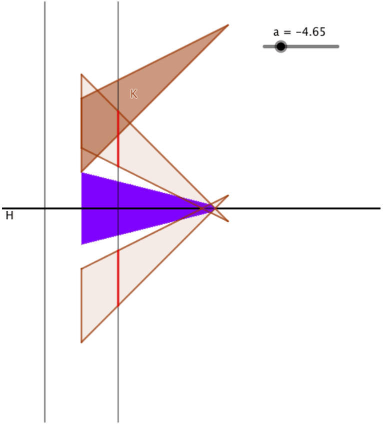

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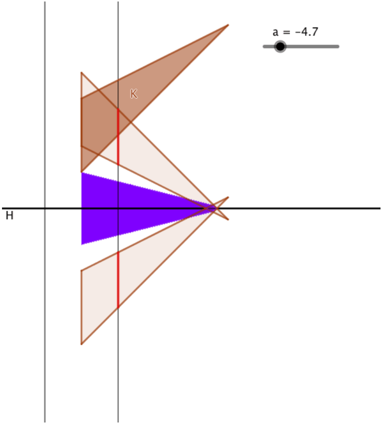

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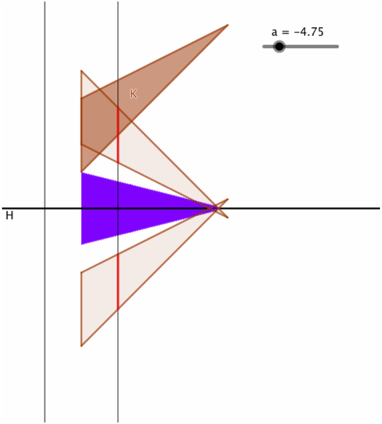

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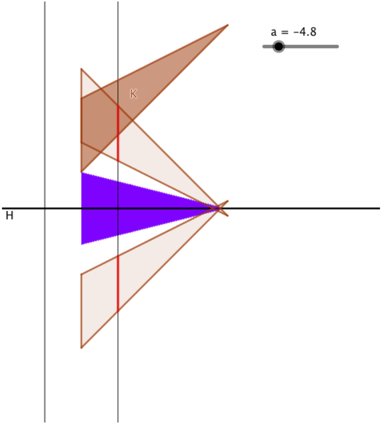

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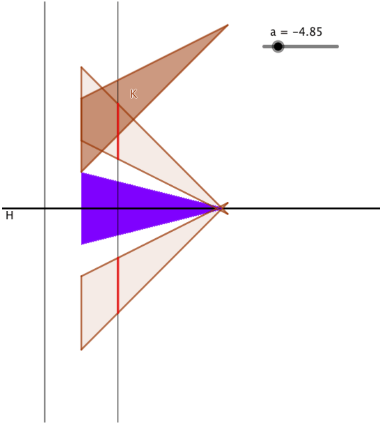

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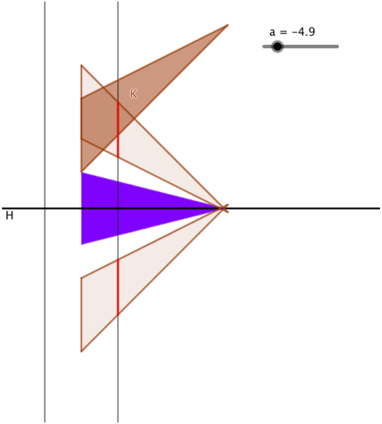

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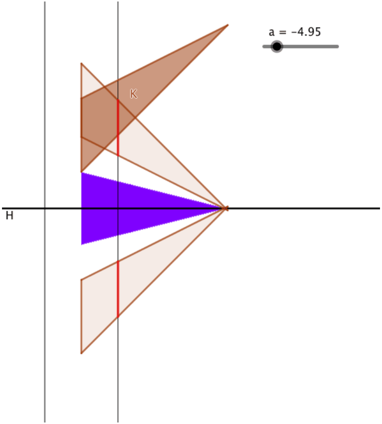

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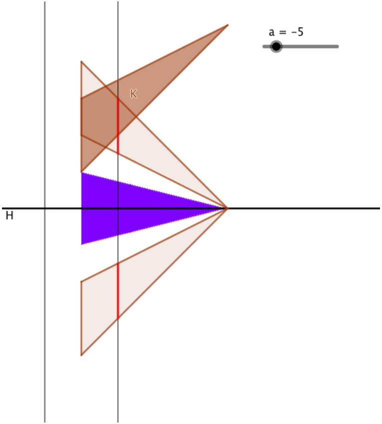

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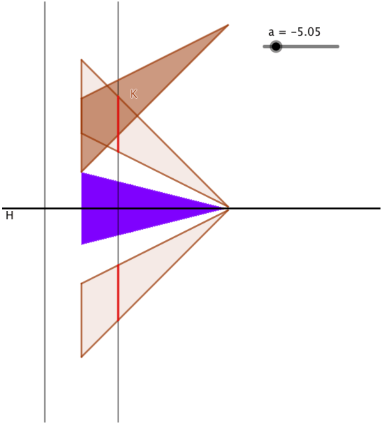

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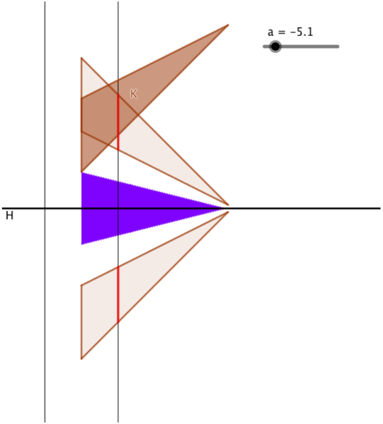

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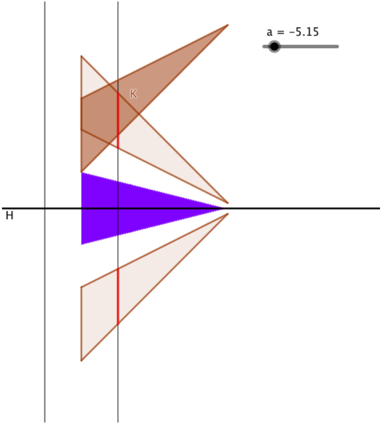

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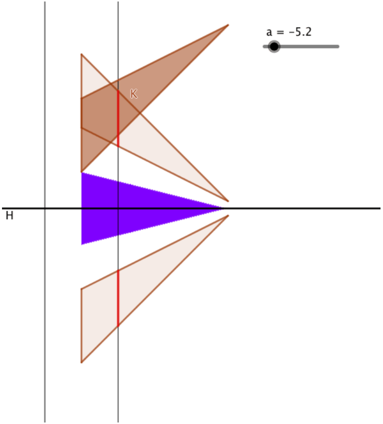

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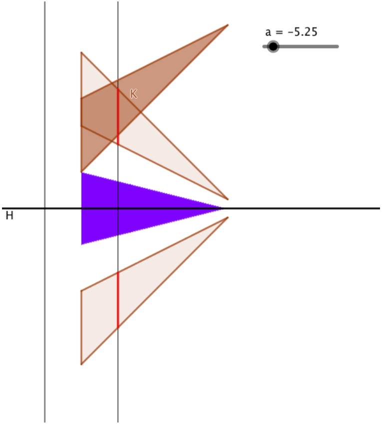

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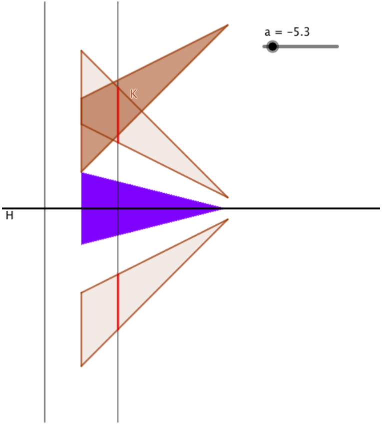

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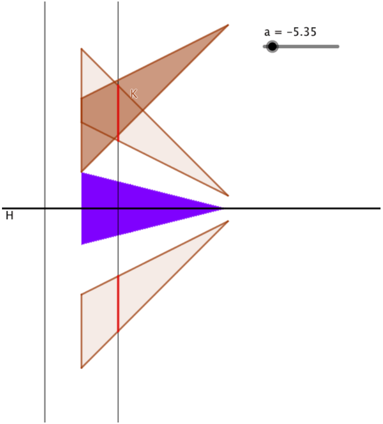

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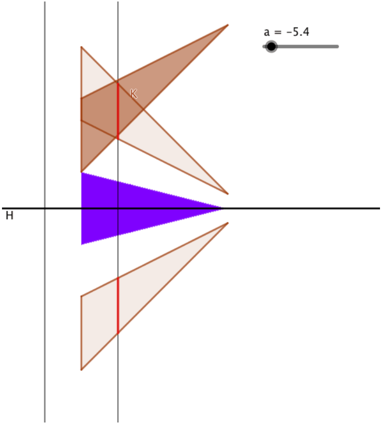

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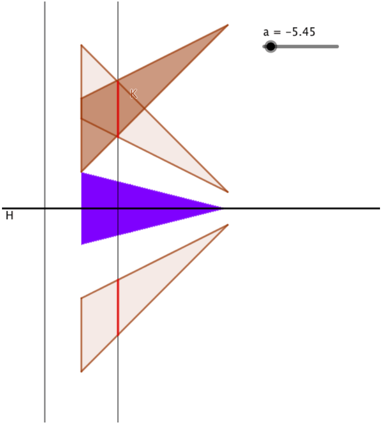

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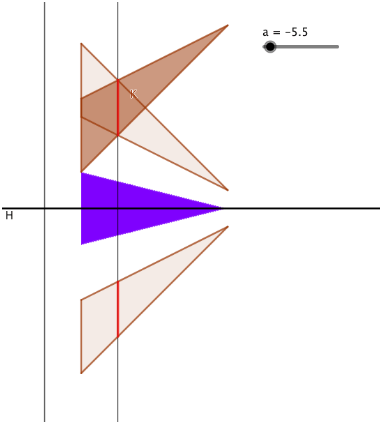

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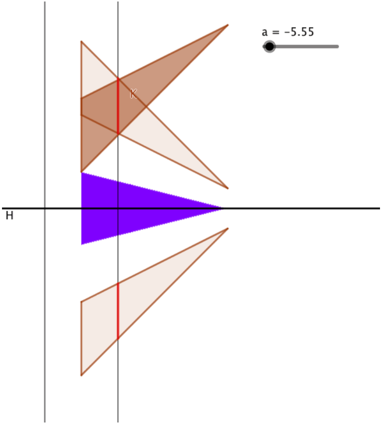

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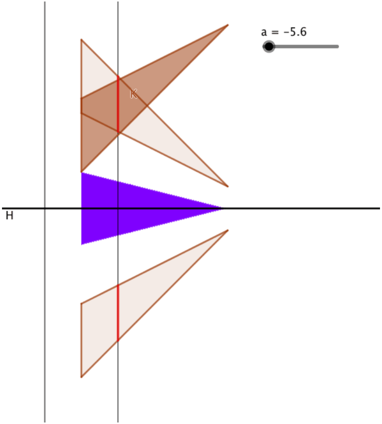

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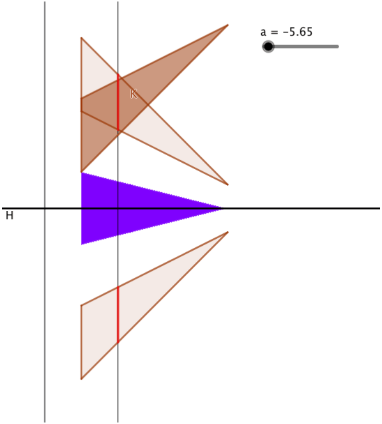

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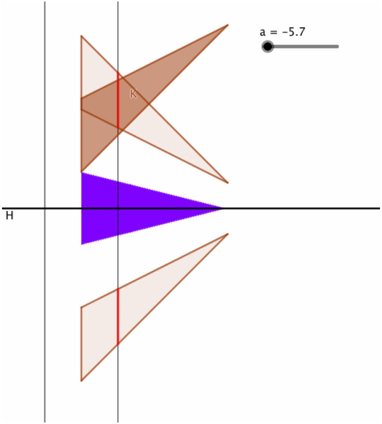

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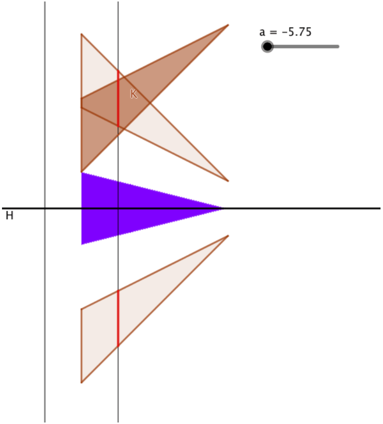

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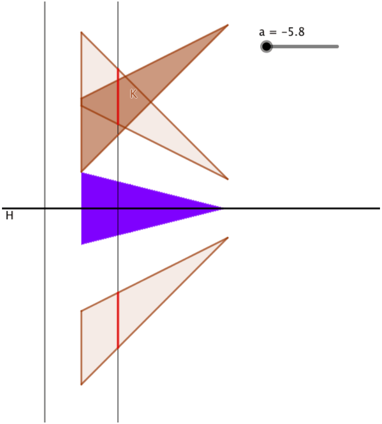

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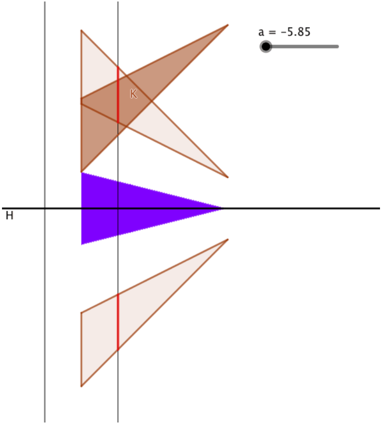

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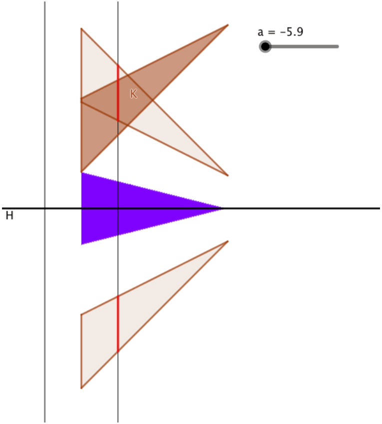

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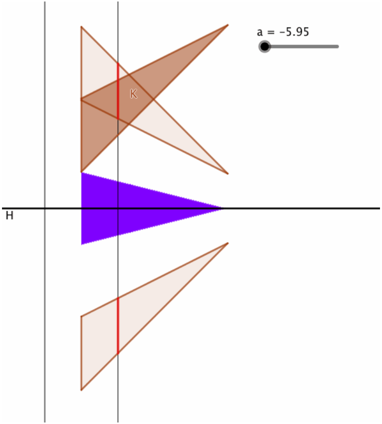

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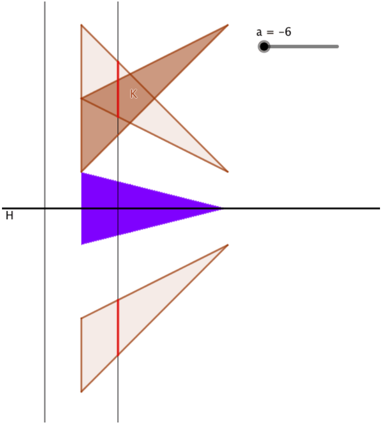

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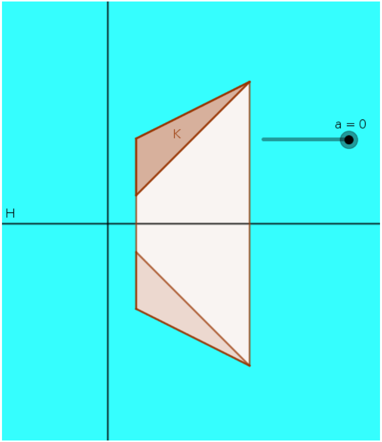

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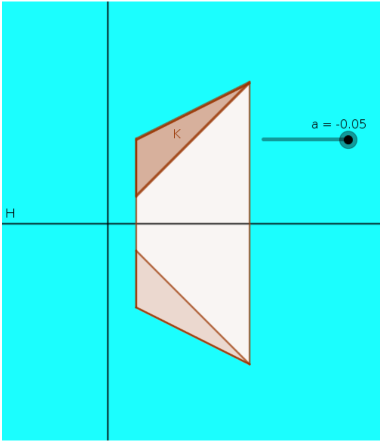

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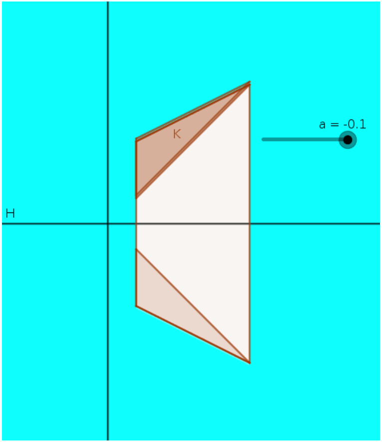

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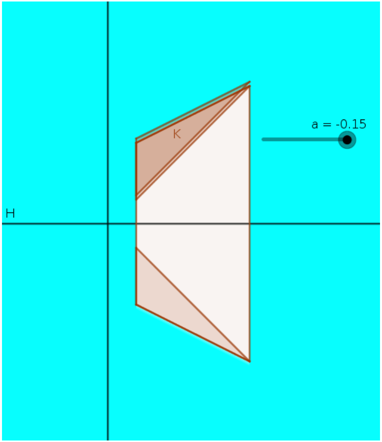

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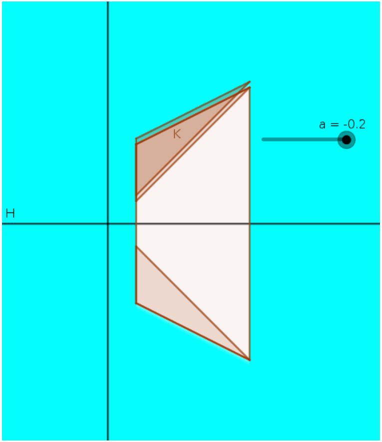

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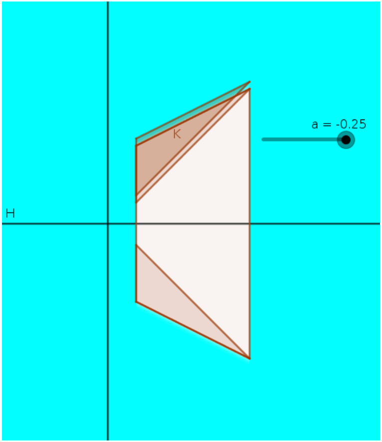

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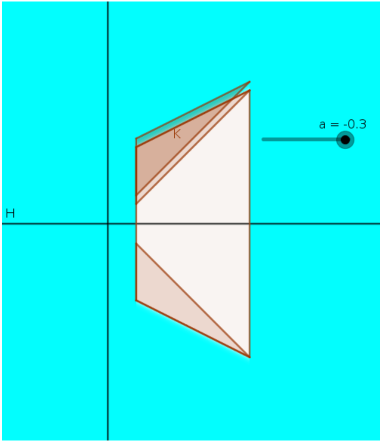

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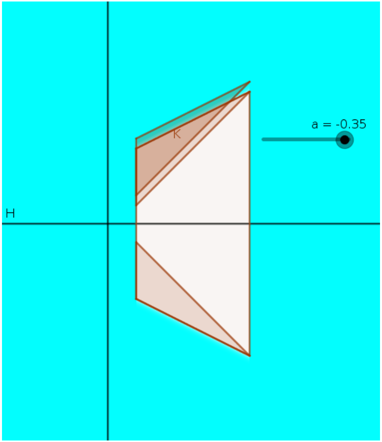

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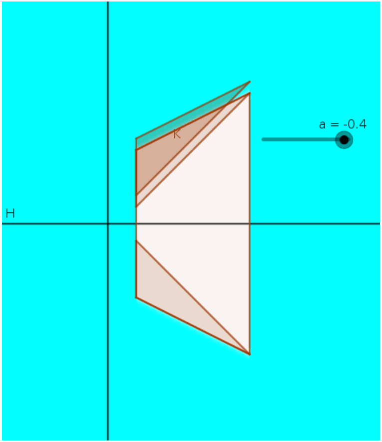

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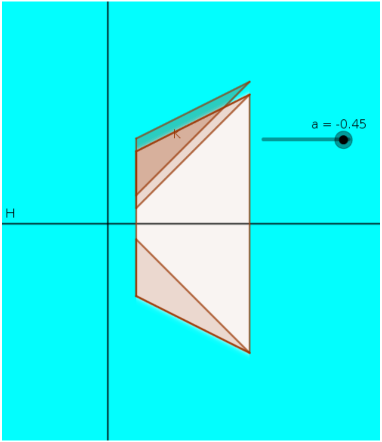

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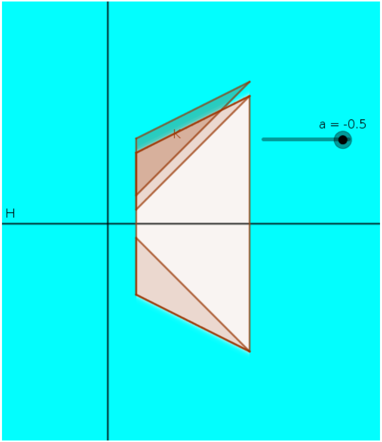

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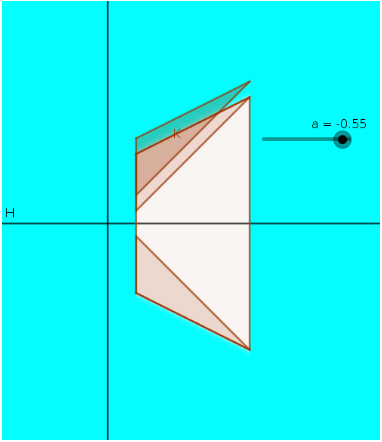

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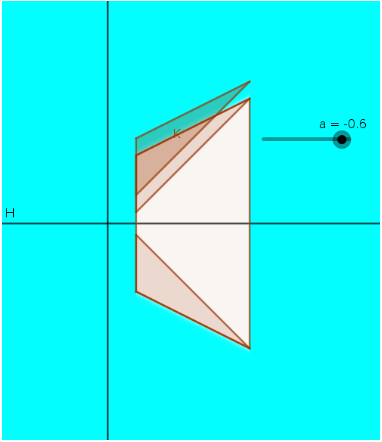

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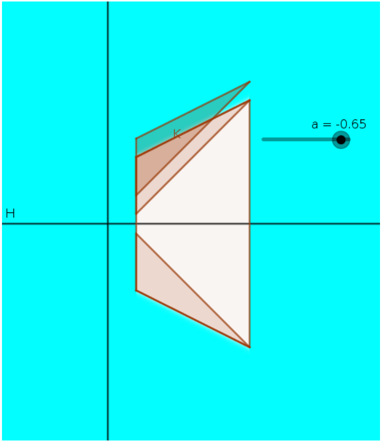

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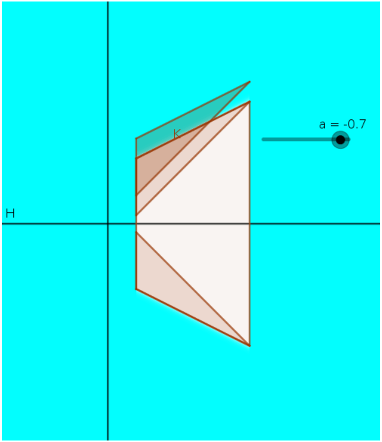

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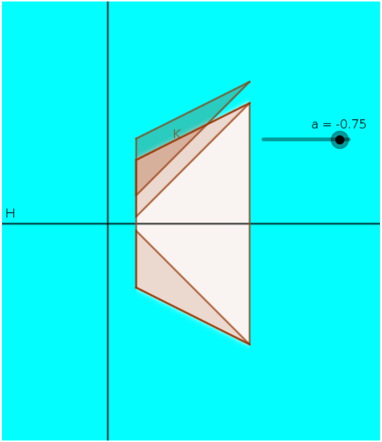

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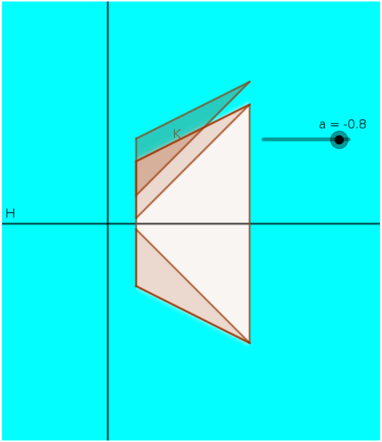

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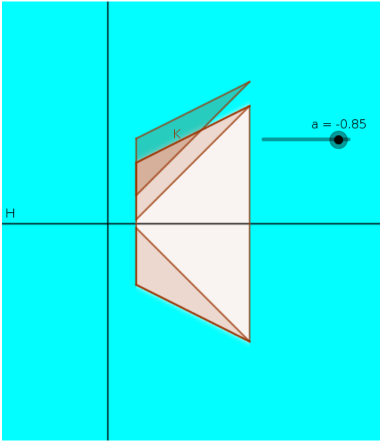

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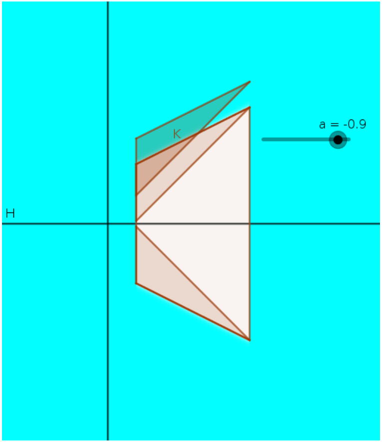

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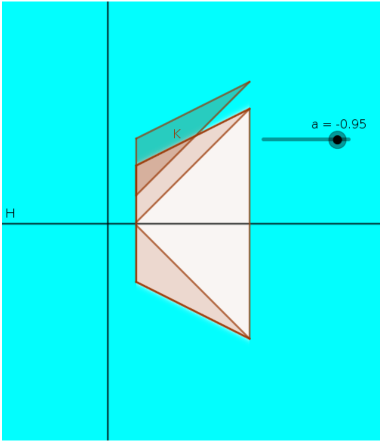

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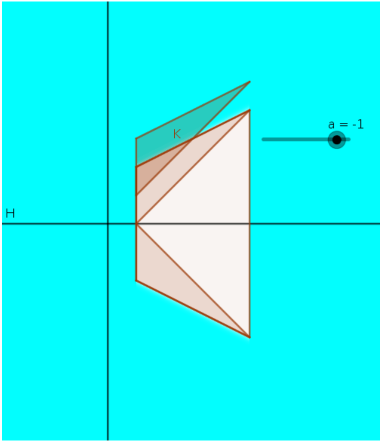

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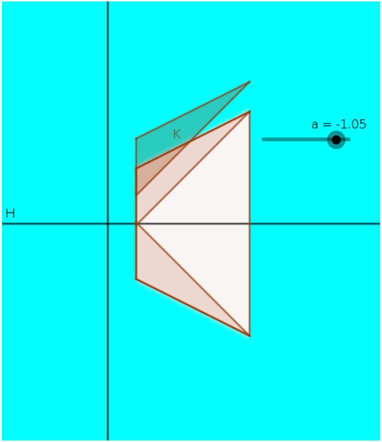

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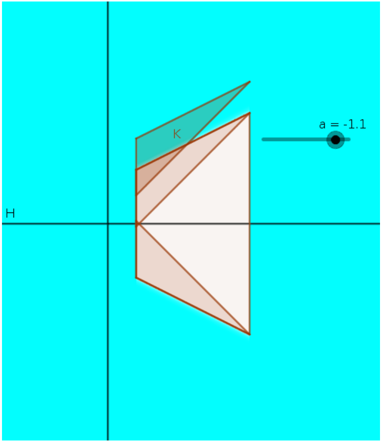

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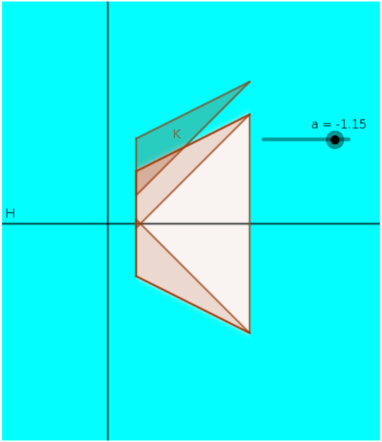

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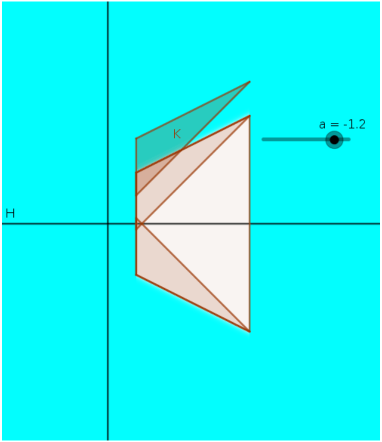

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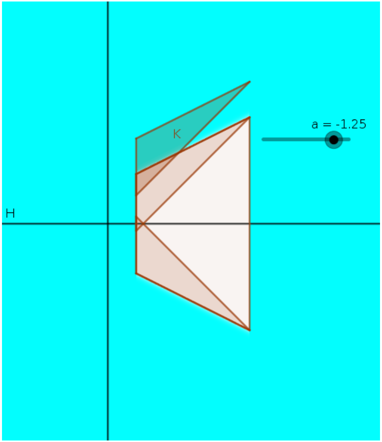

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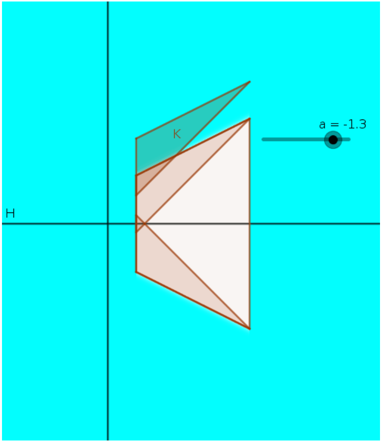

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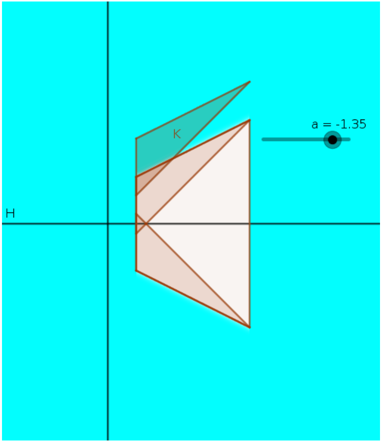

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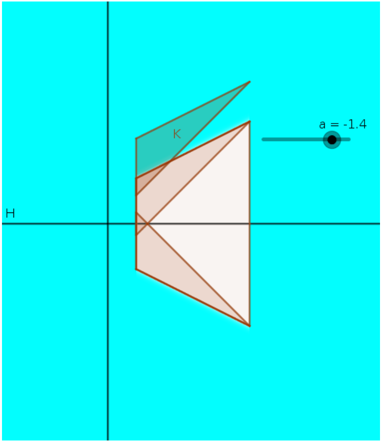

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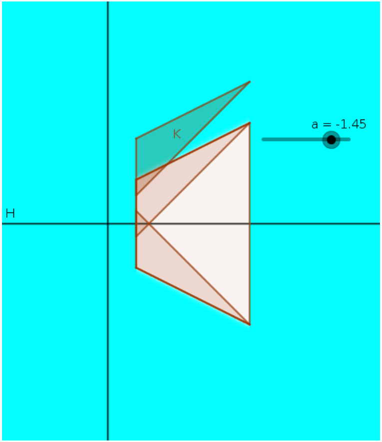

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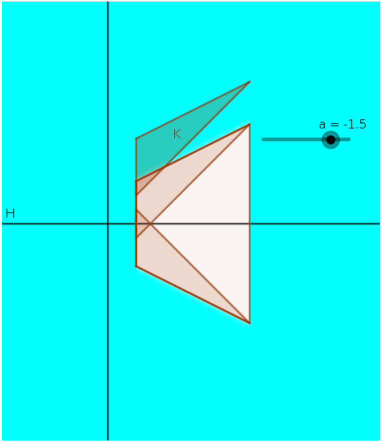

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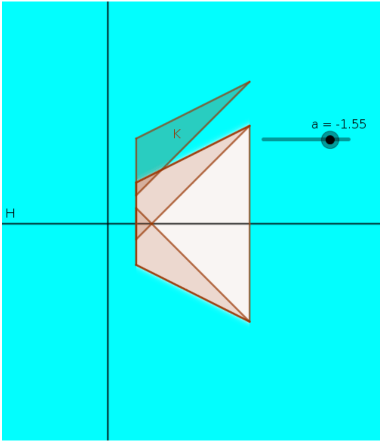

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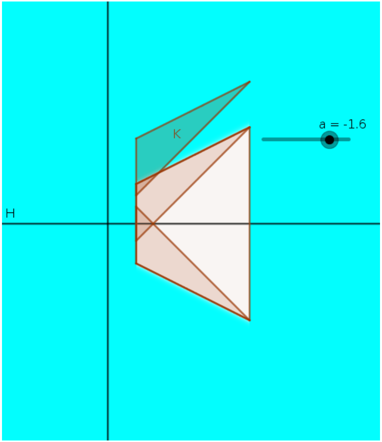

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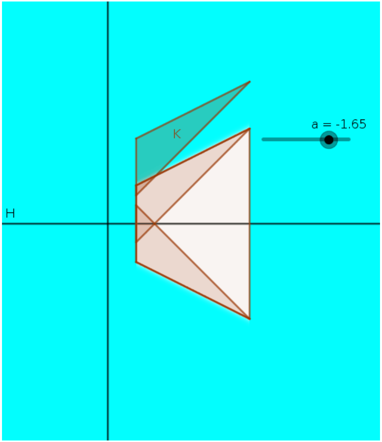

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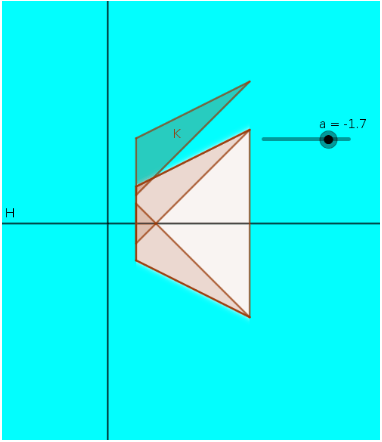

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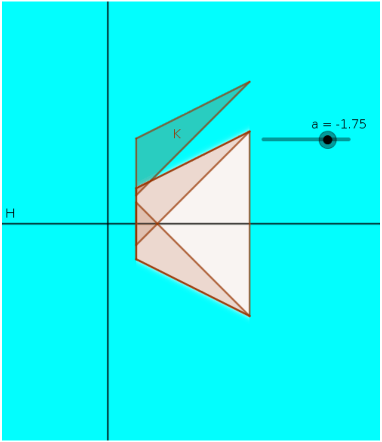

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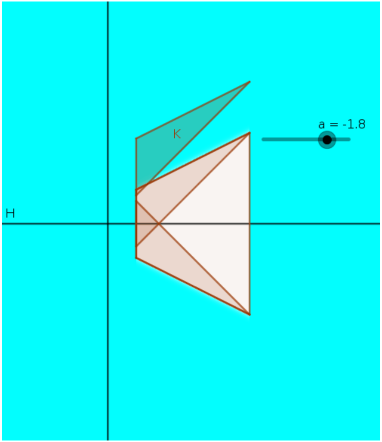

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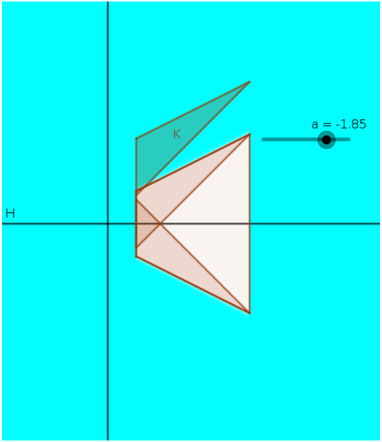

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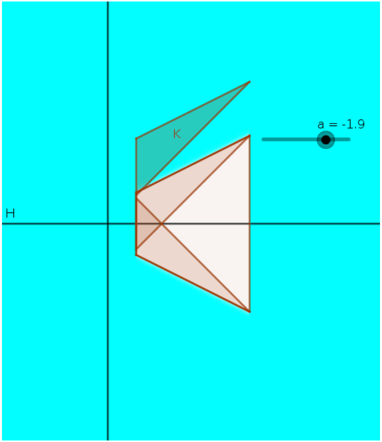

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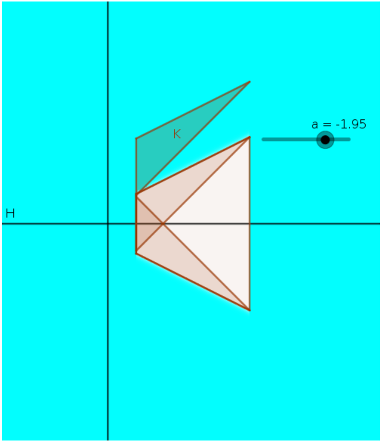

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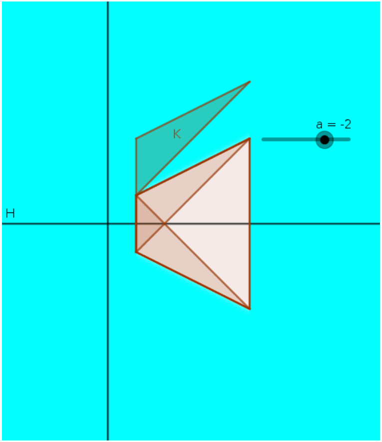

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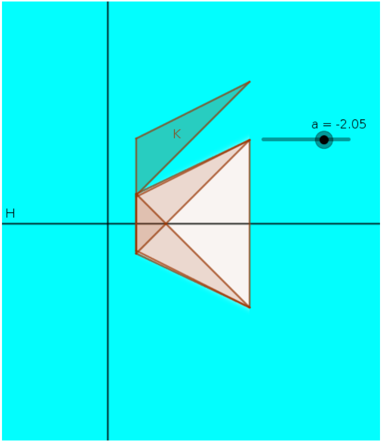

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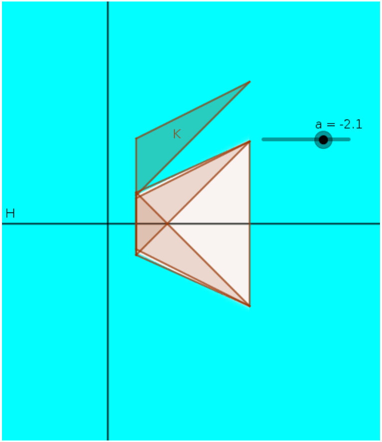

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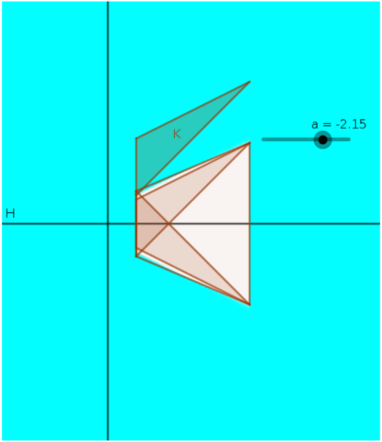

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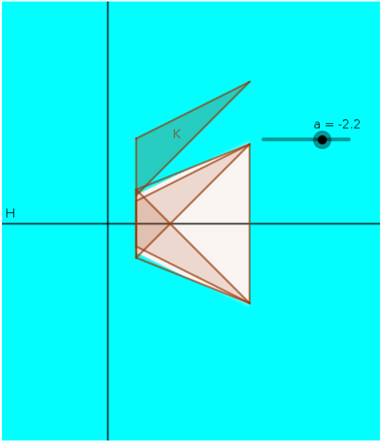

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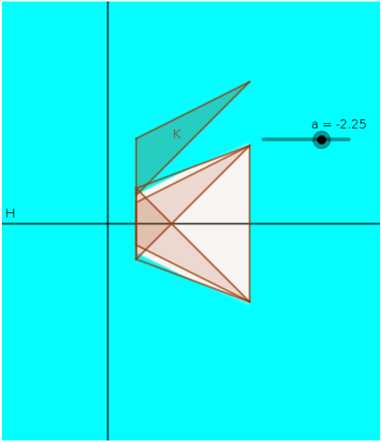

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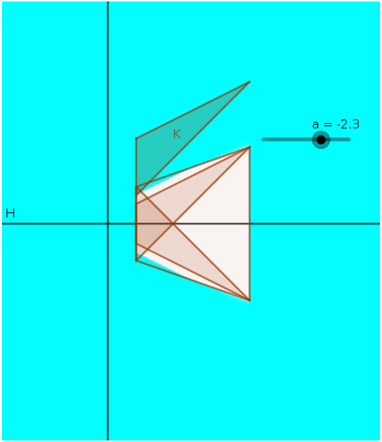

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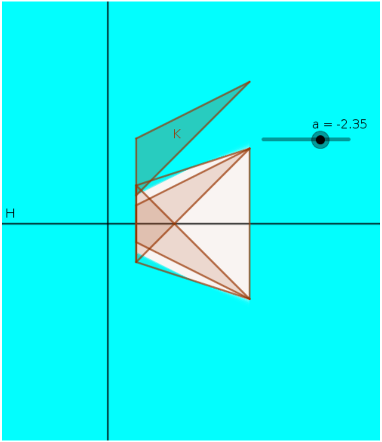

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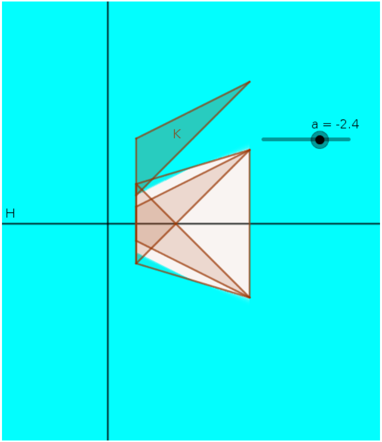

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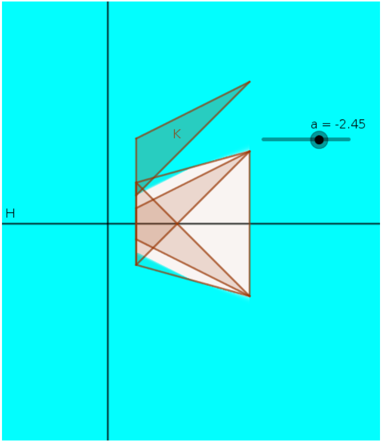

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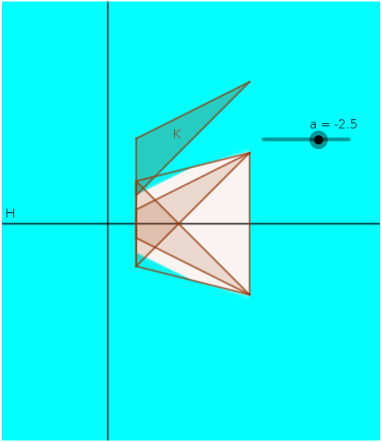

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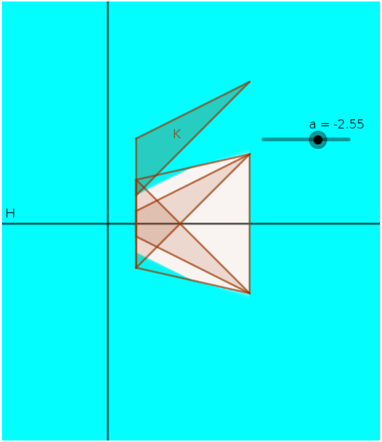

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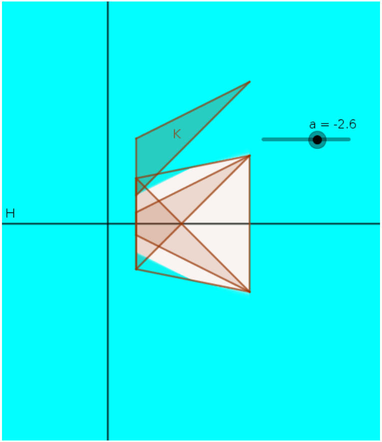

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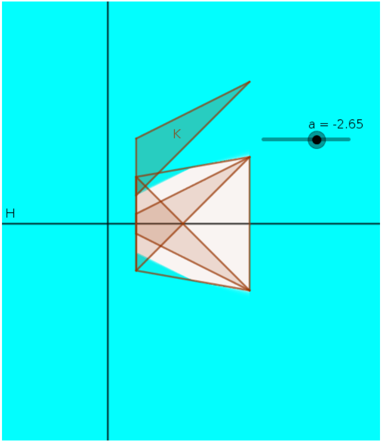

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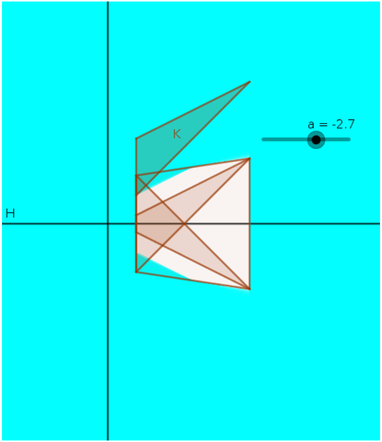

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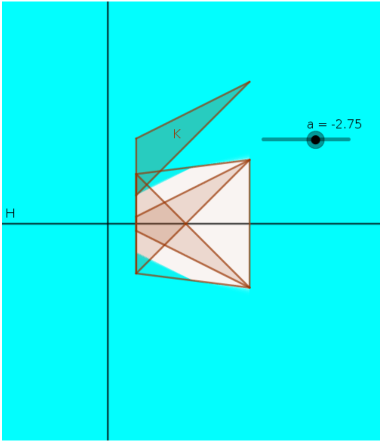

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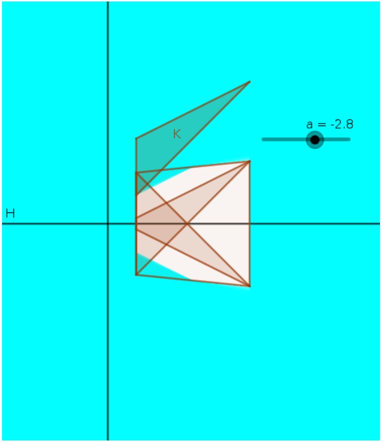

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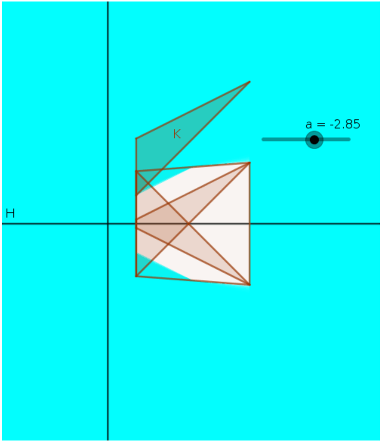

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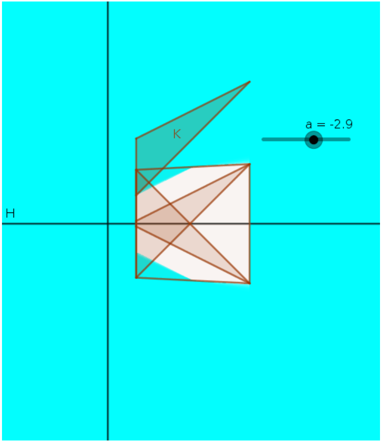

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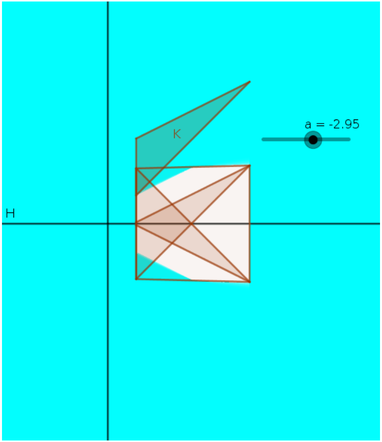

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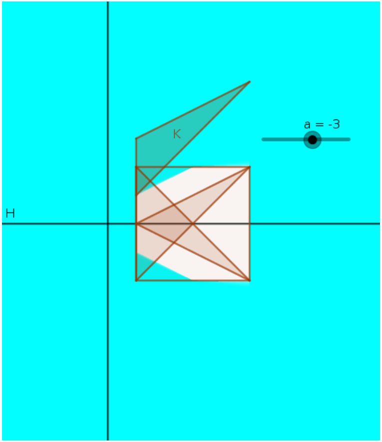

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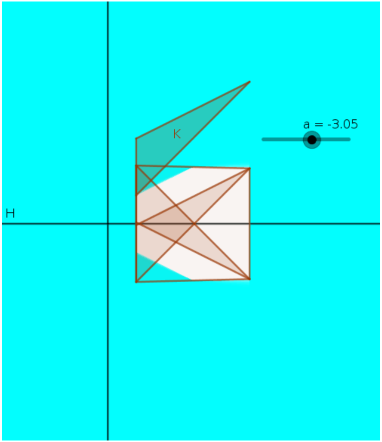

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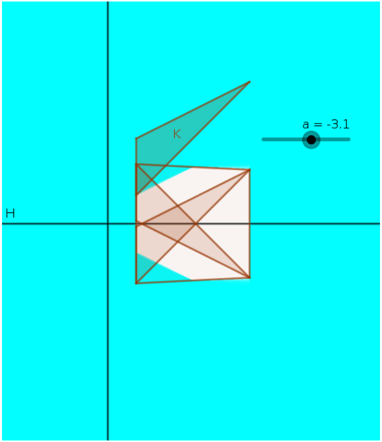

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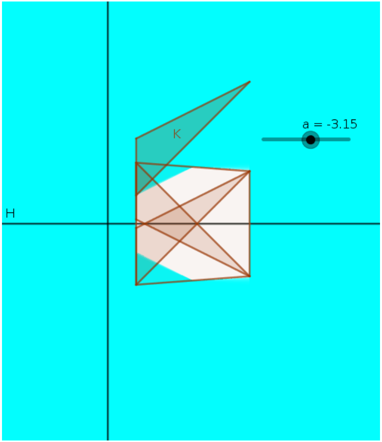

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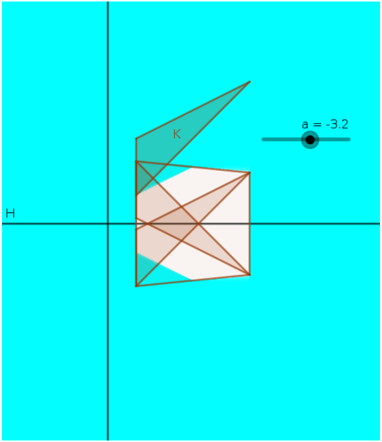

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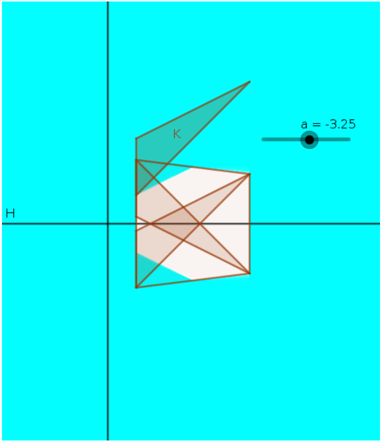

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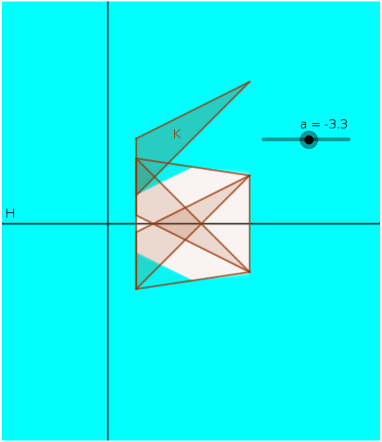

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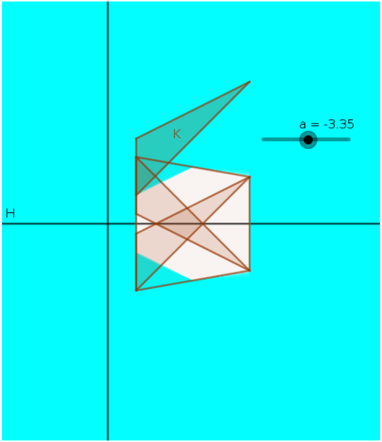

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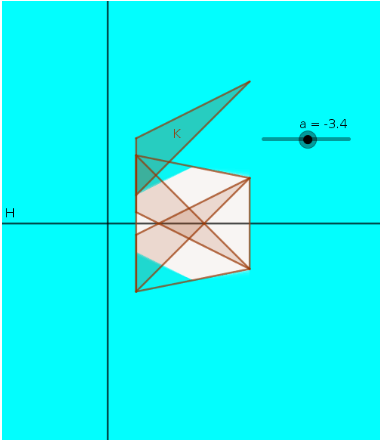

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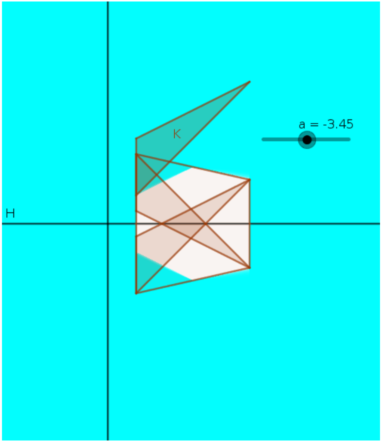

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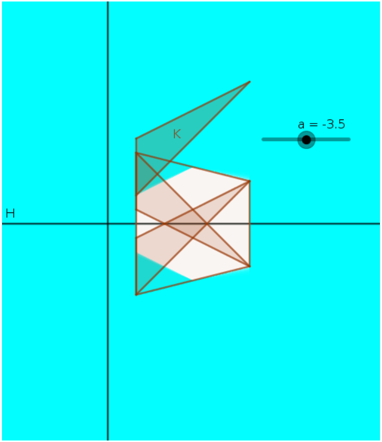

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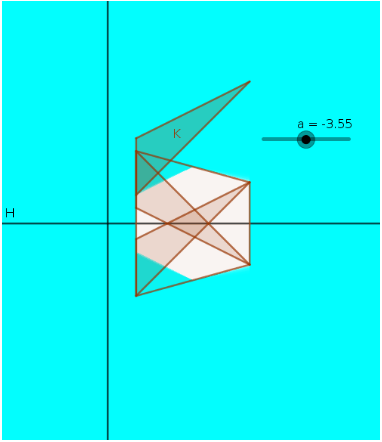

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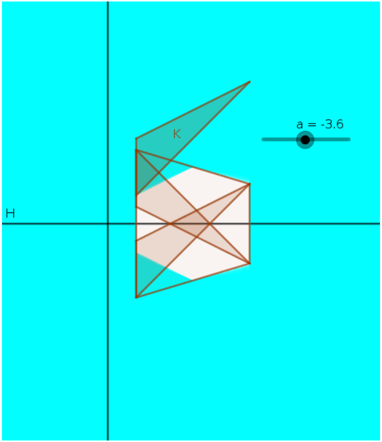

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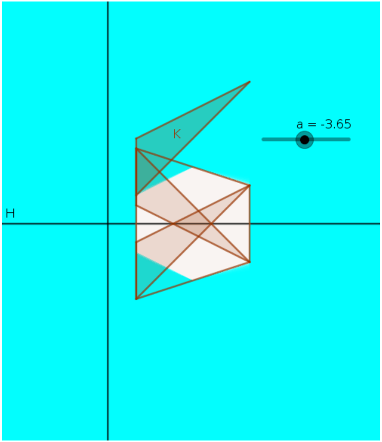

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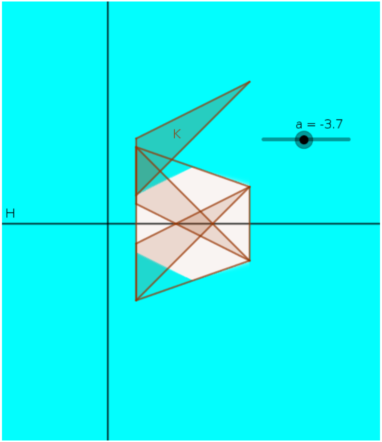

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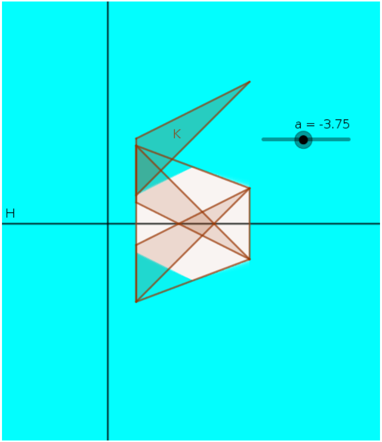

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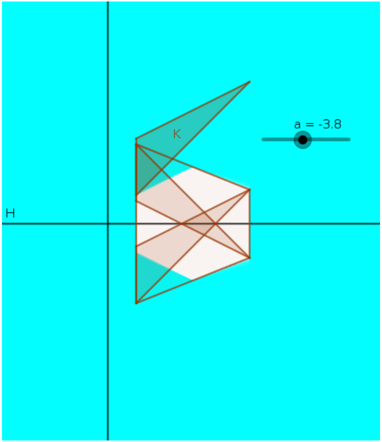

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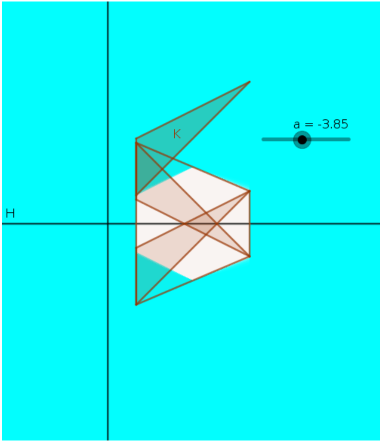

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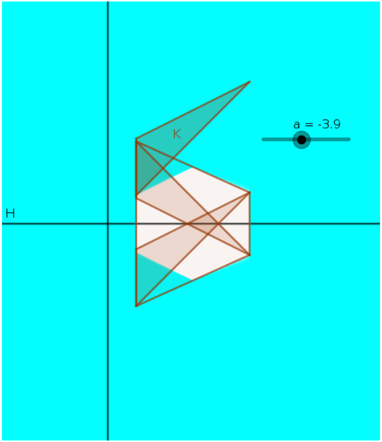

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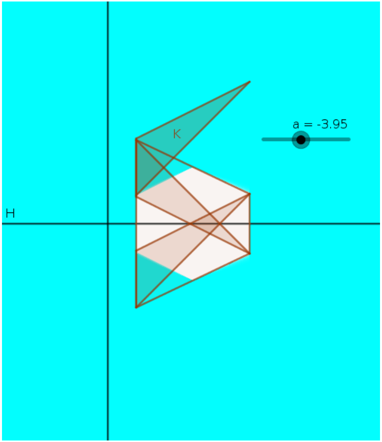

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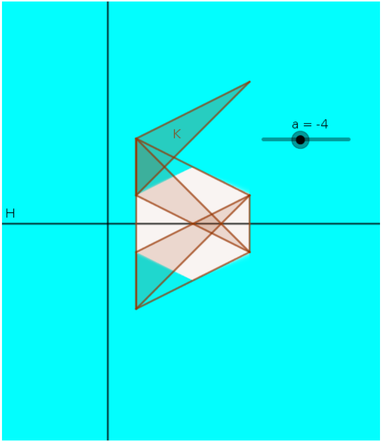

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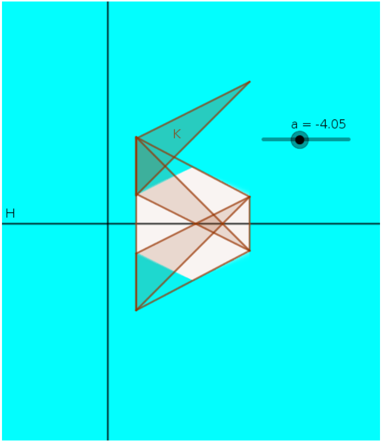

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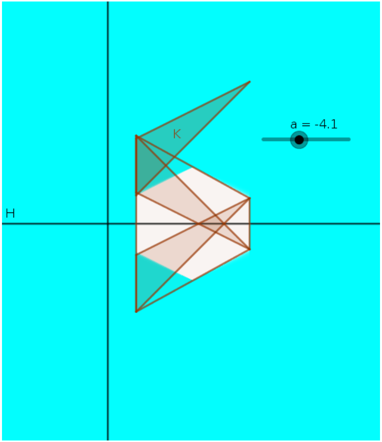

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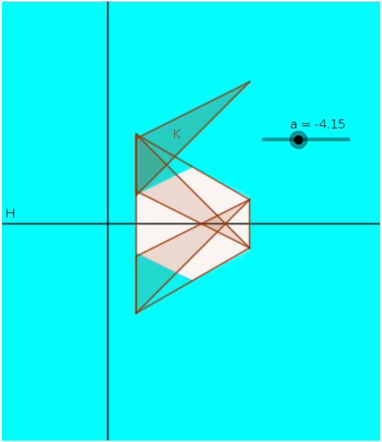

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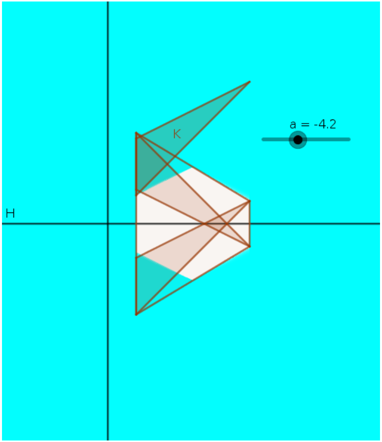

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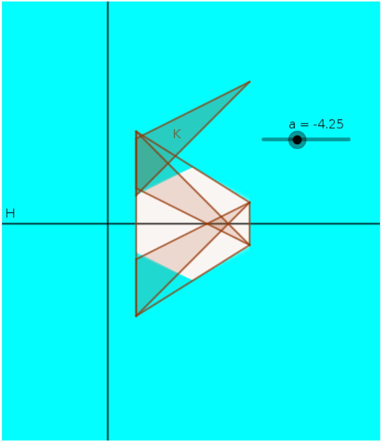

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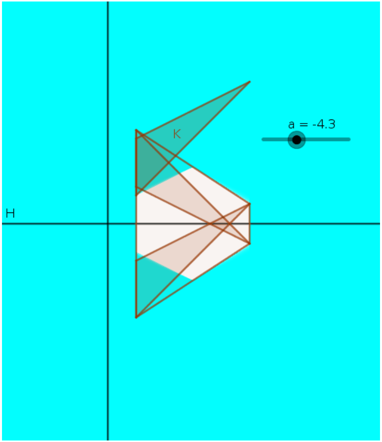

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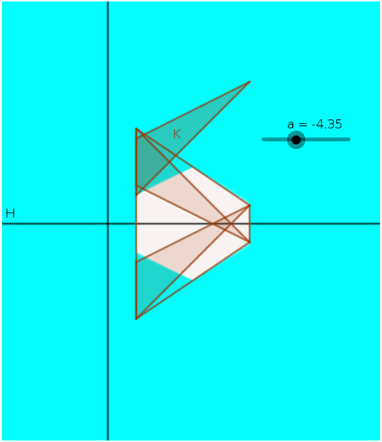

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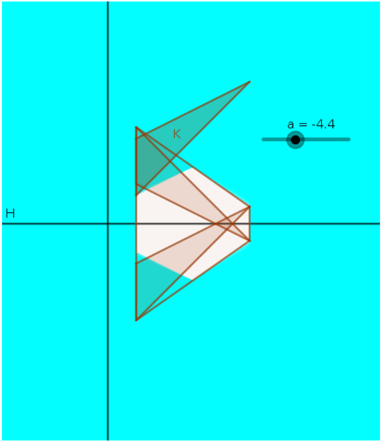

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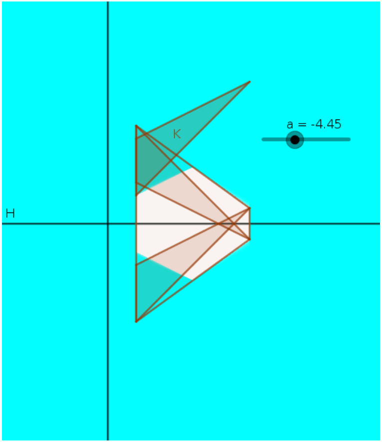

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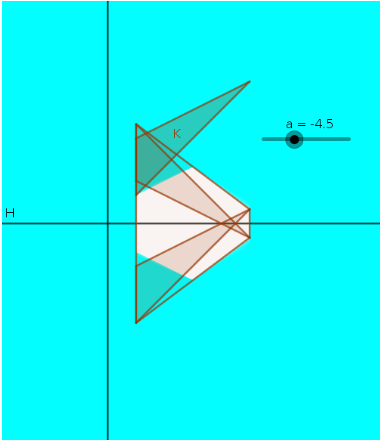

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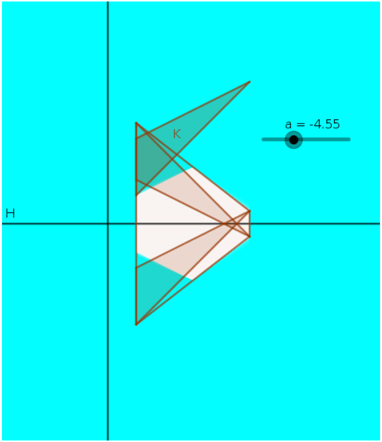

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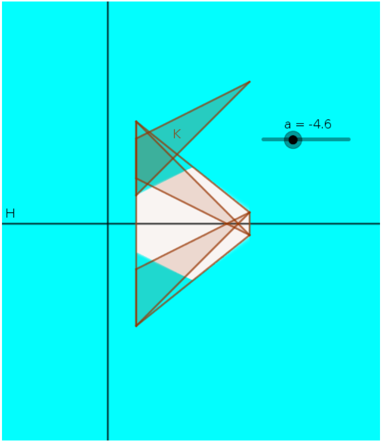

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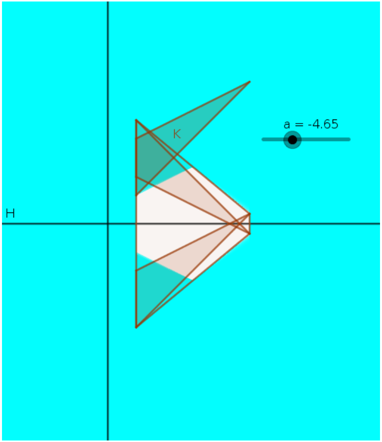

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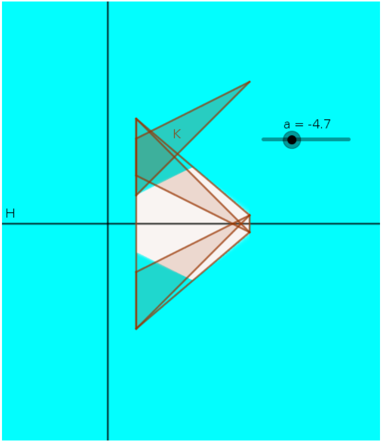

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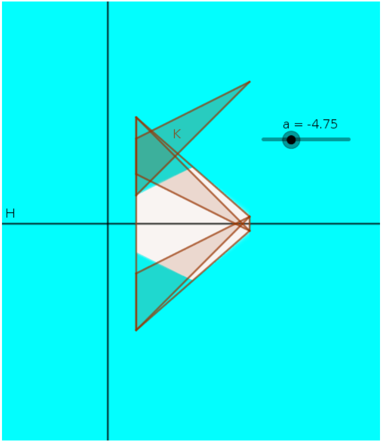

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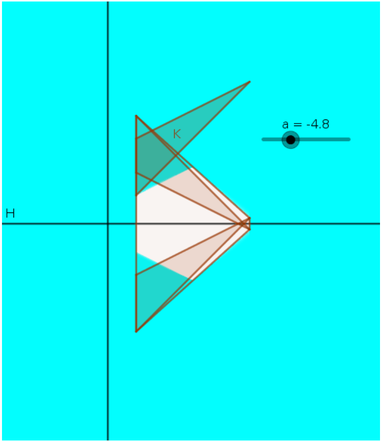

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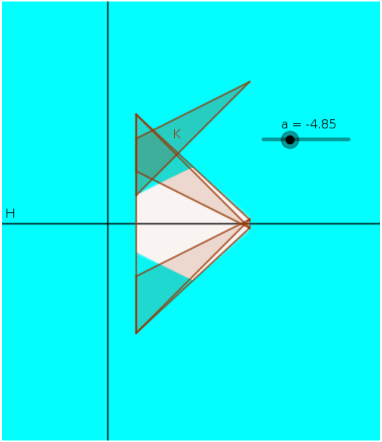

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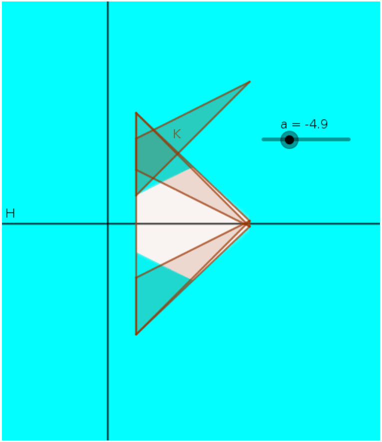

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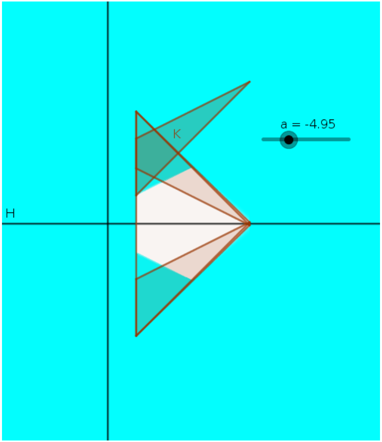

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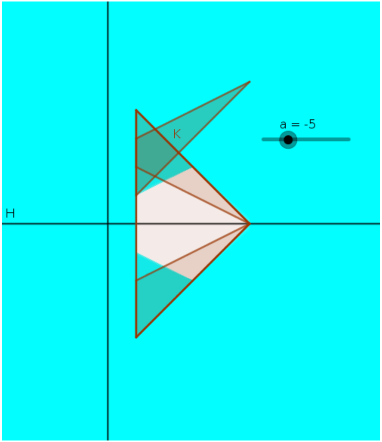

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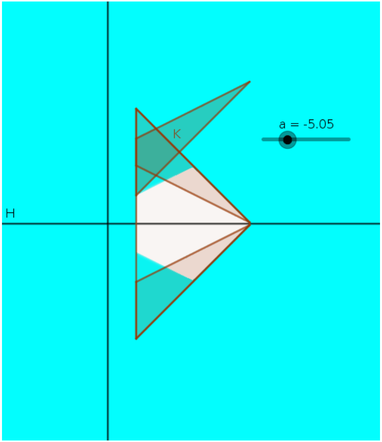

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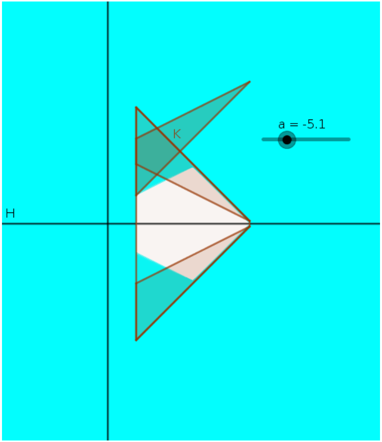

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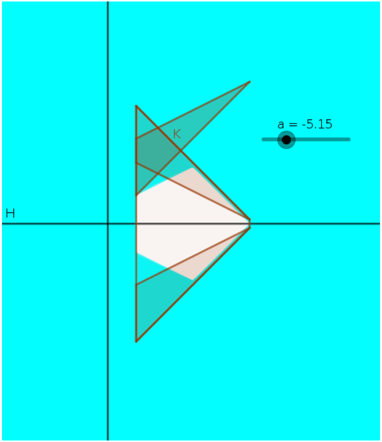

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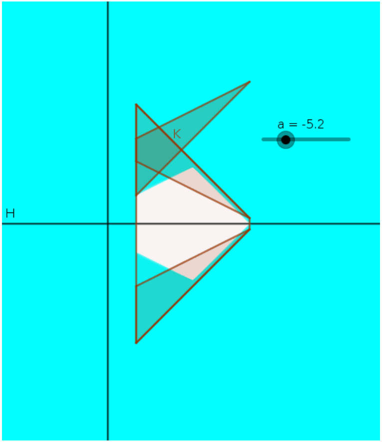

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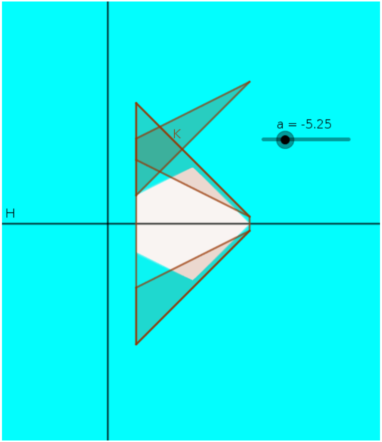

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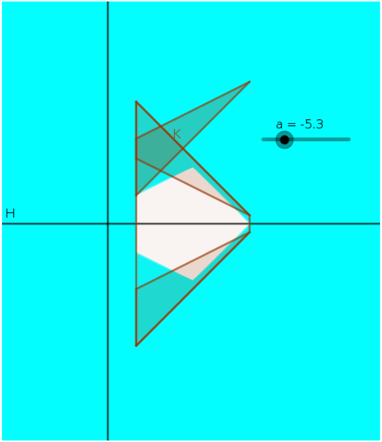

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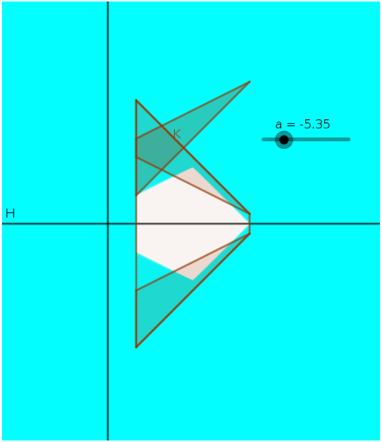

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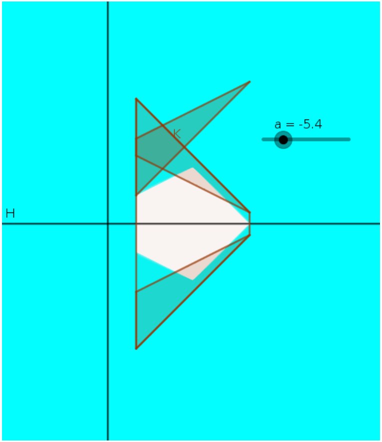

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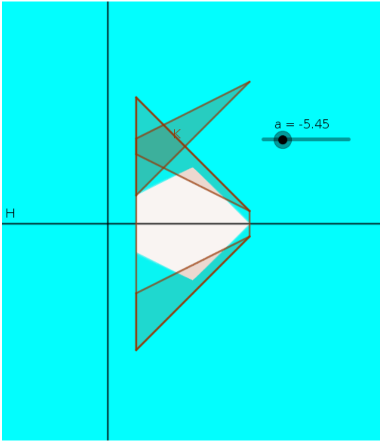

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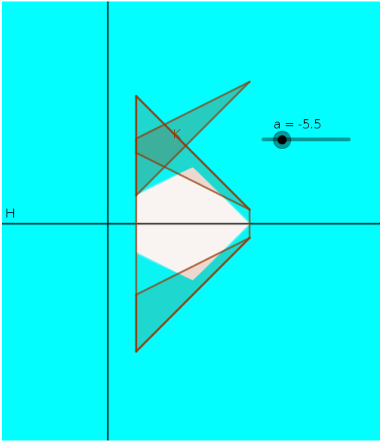

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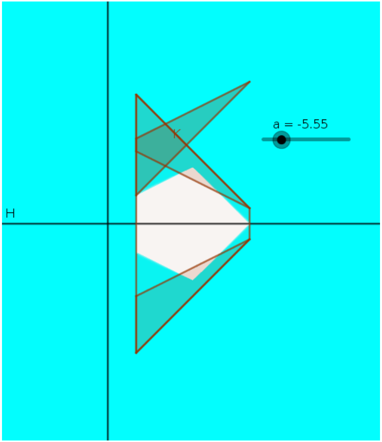

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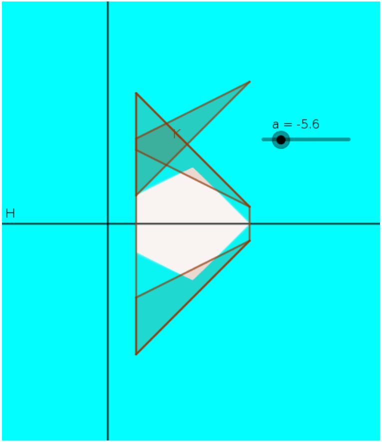

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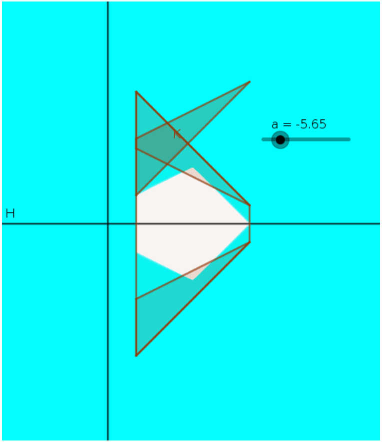

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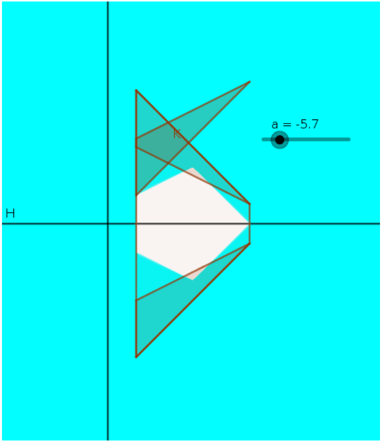

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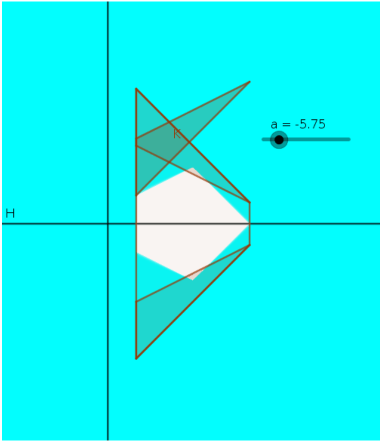

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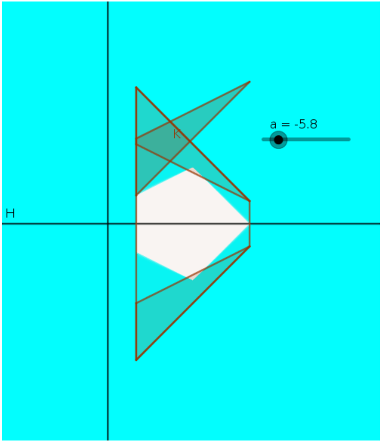

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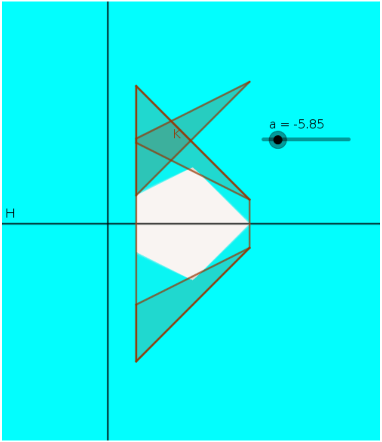

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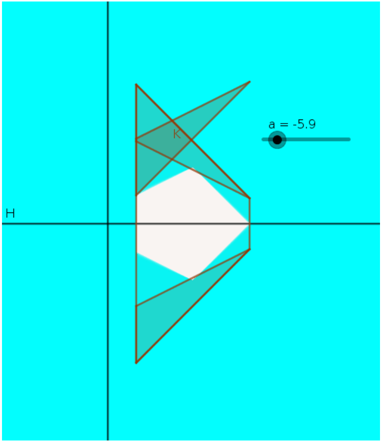

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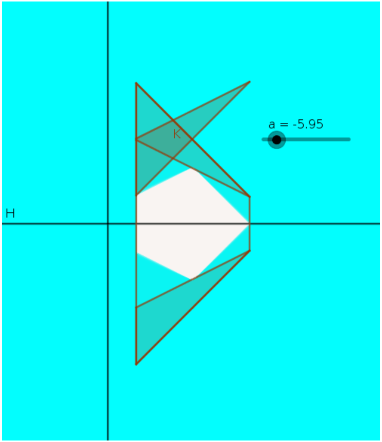

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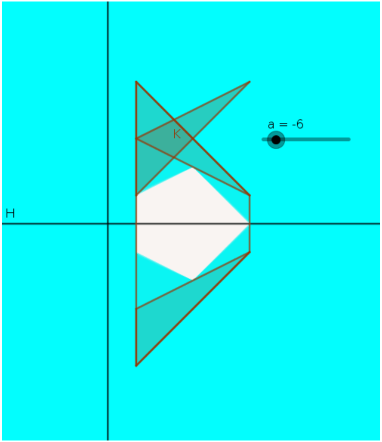

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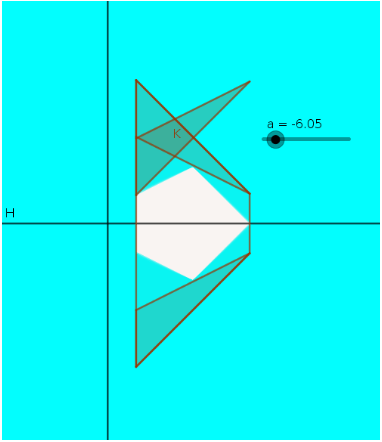

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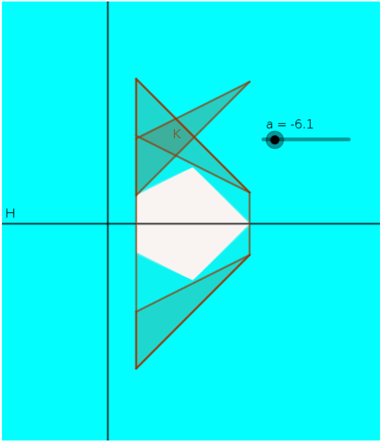

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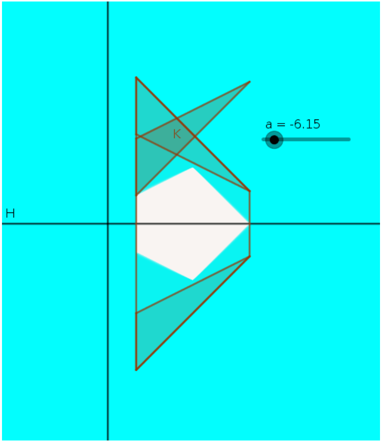

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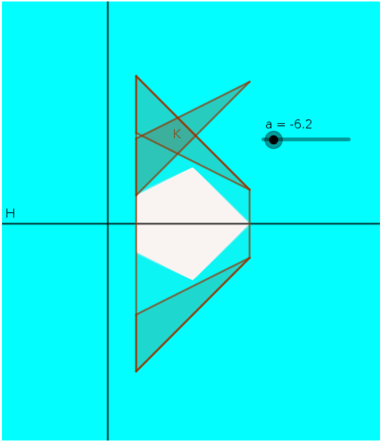

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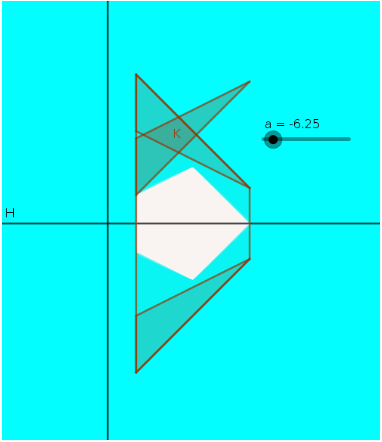

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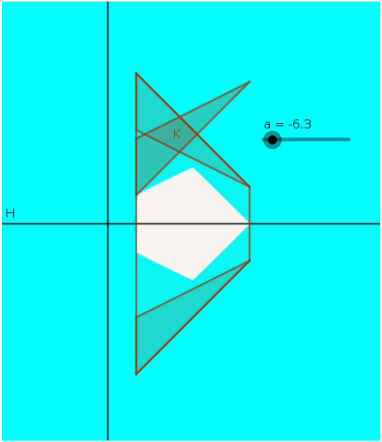

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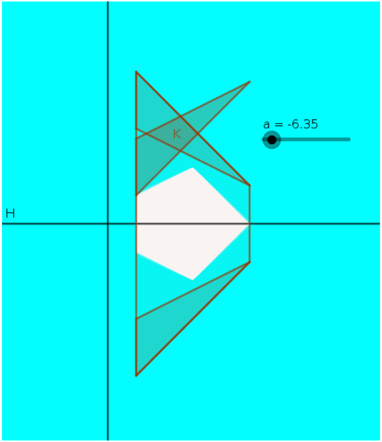

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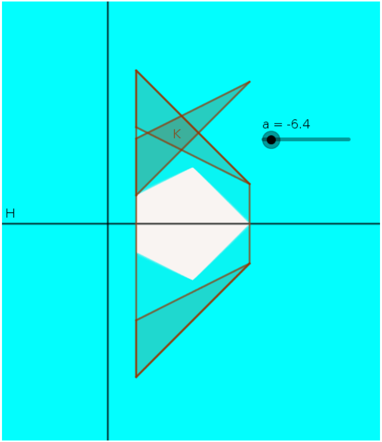

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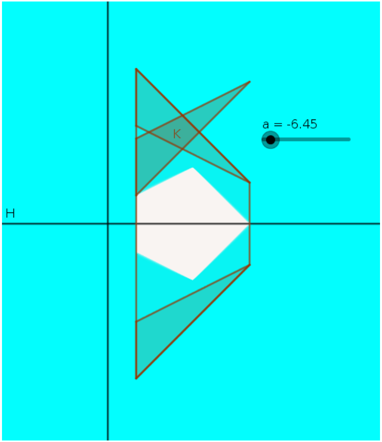

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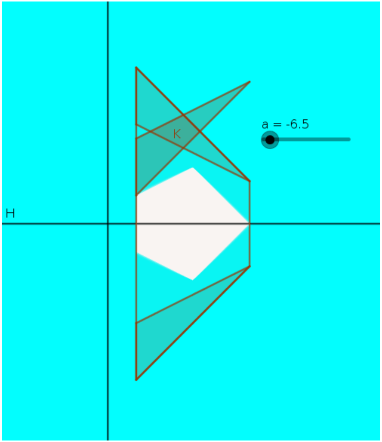

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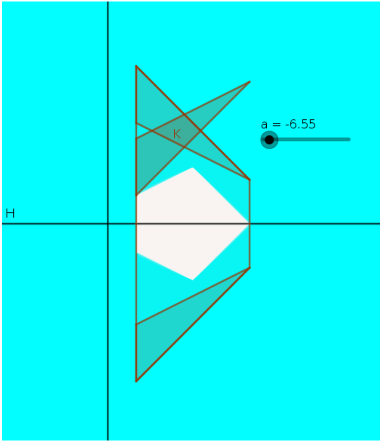

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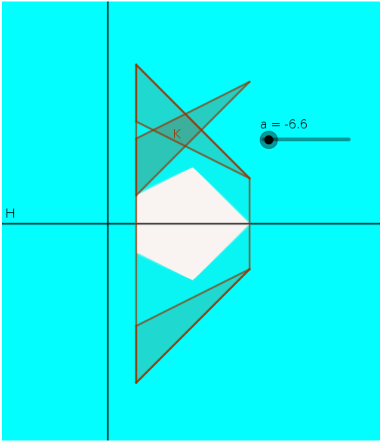

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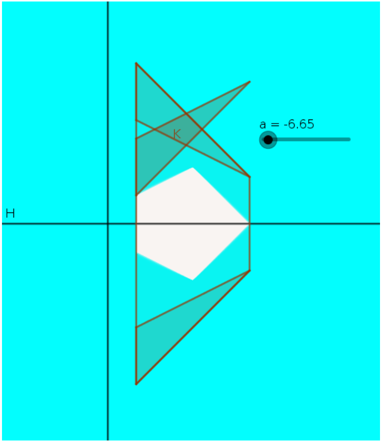

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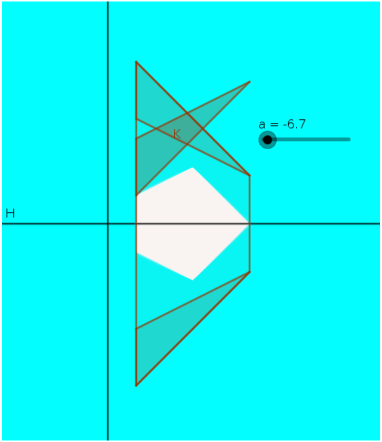

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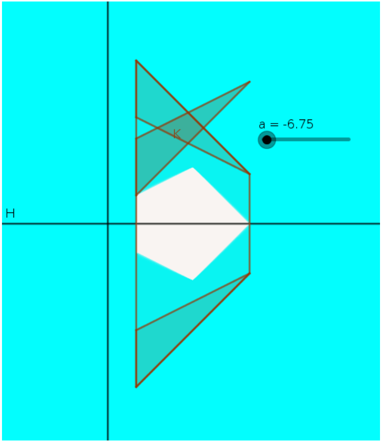

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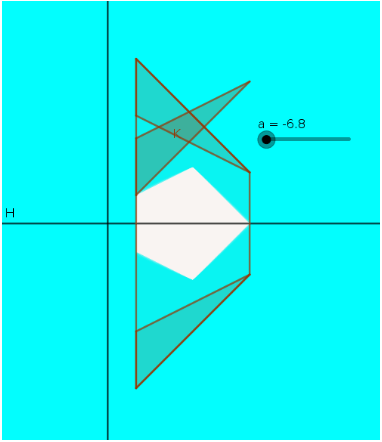

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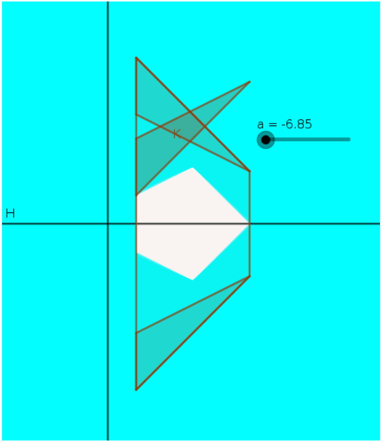

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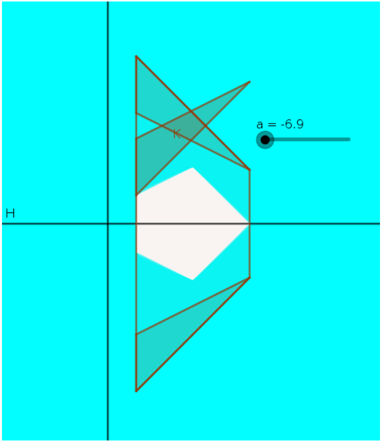

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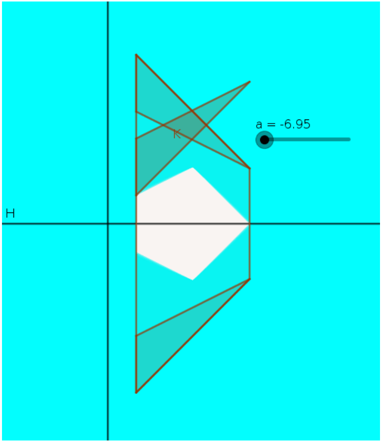

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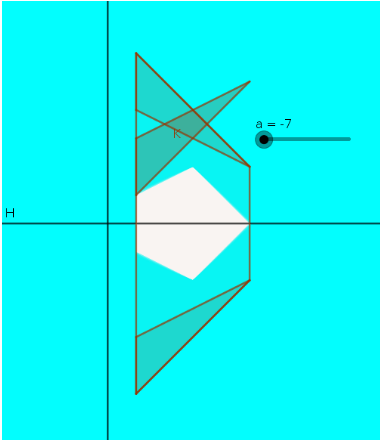

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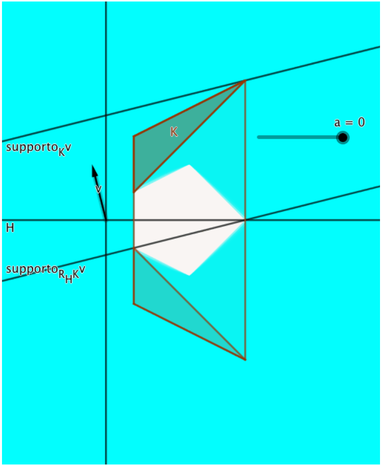

Let $v \in S^{n-1}$. For the support functions we have

$$h_{\bigcap_y conv\left((K+y), R_H(K+y)\right)}(v) = \inf_y \left(h_{conv\left((K+y), R_H(K+y)\right)}(v)\right) = \inf_y \left(\max\left(h_{K+y}(v), h_{R_H(K+y)}(v)\right)\right).$$

The animation shows that this infimum is attained when $h_{K+y}(v) = h_{R_H(K+y)}(v)$, and this happens exactly for y such that

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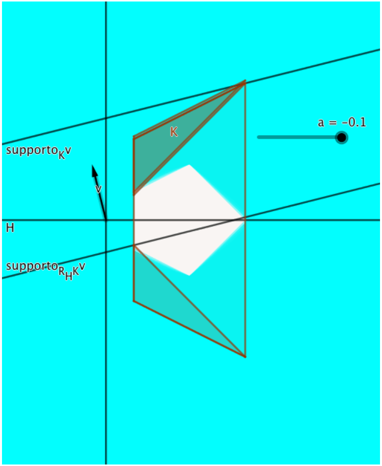

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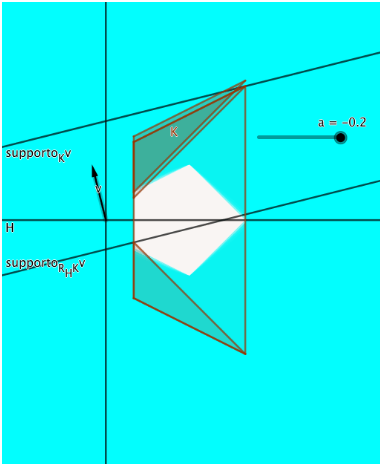

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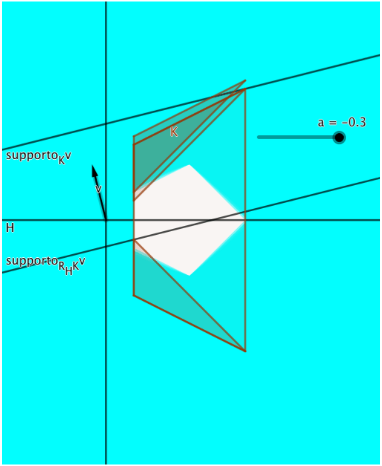

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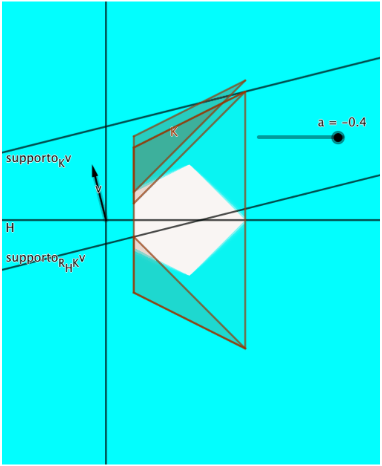

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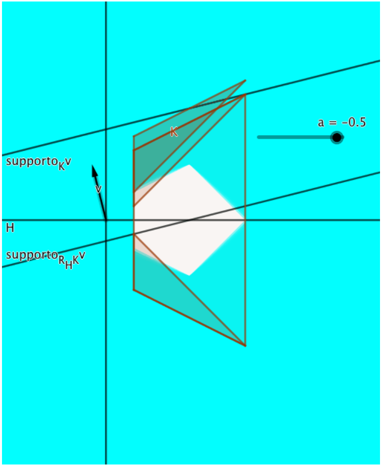

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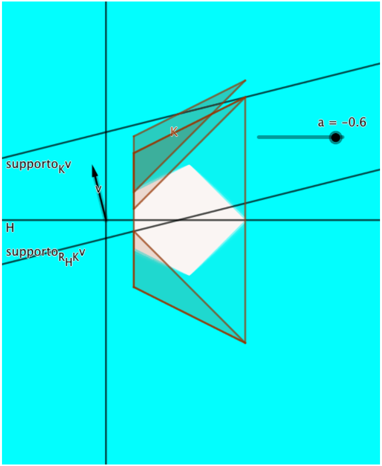

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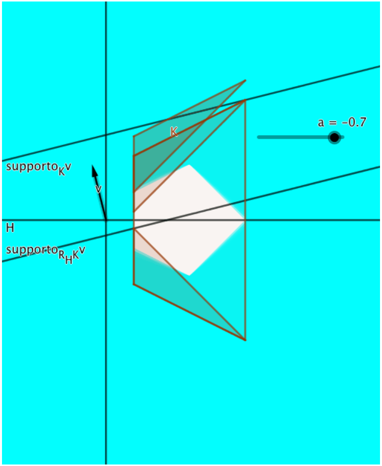

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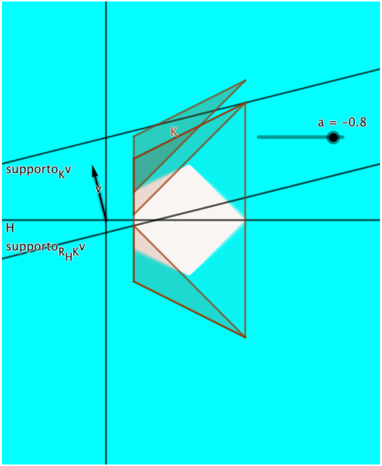

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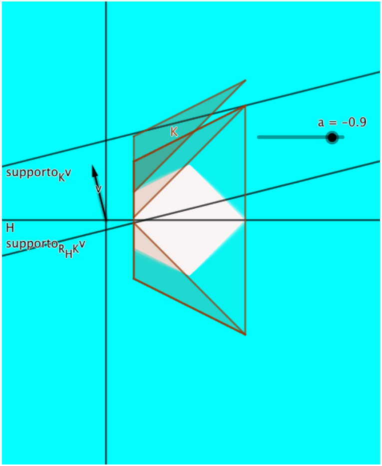

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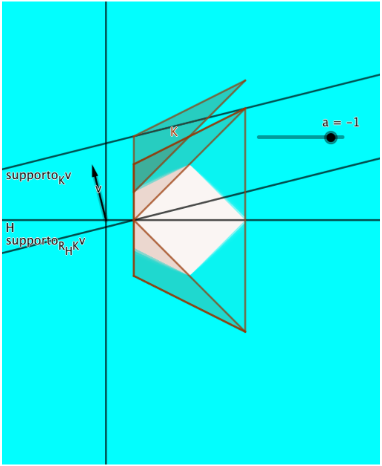

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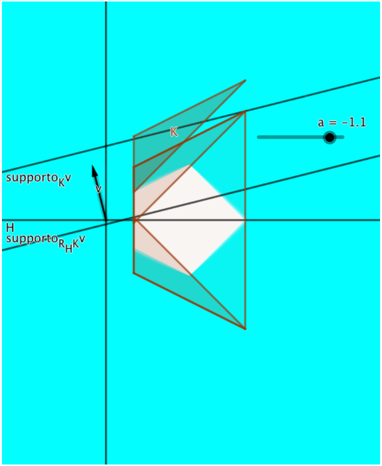

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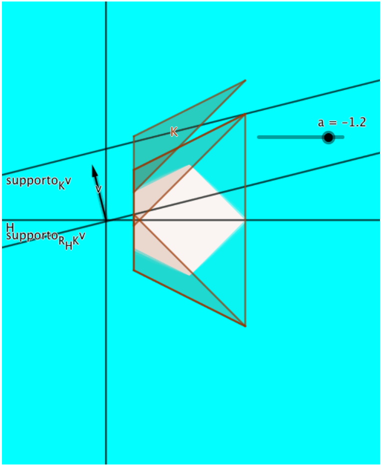

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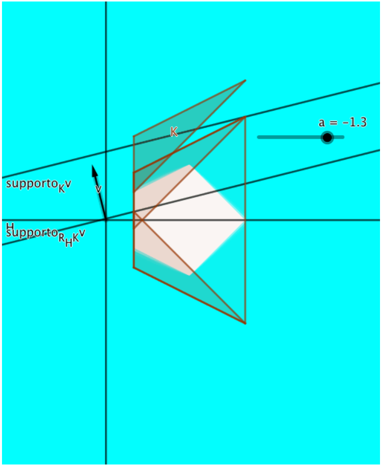

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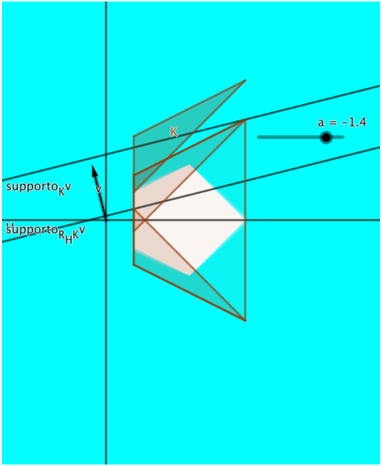

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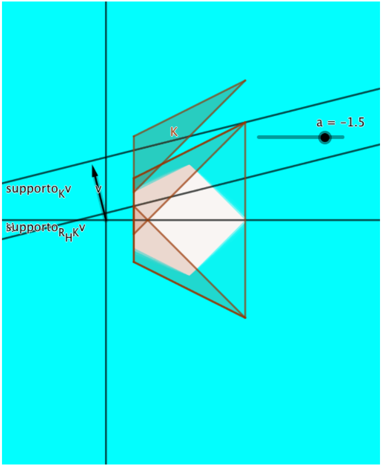

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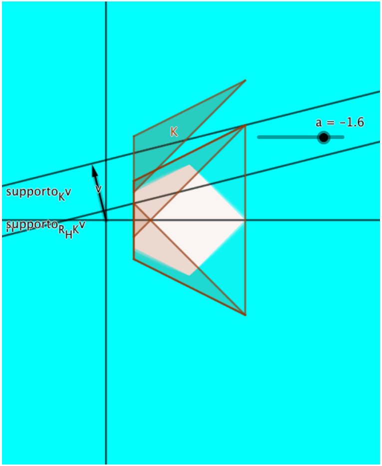

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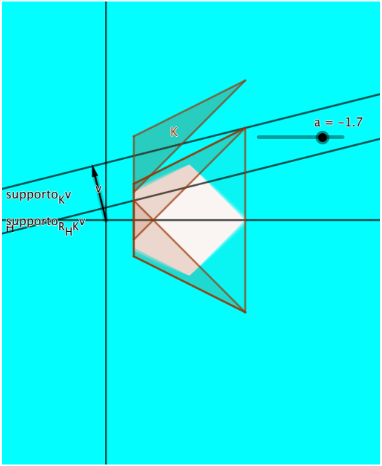

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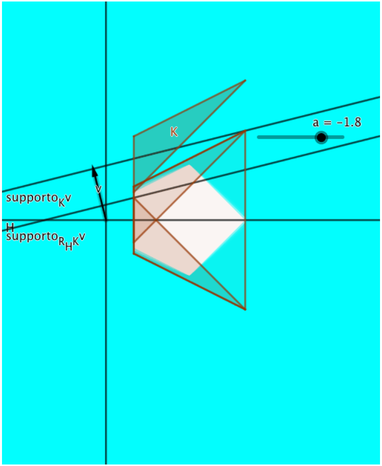

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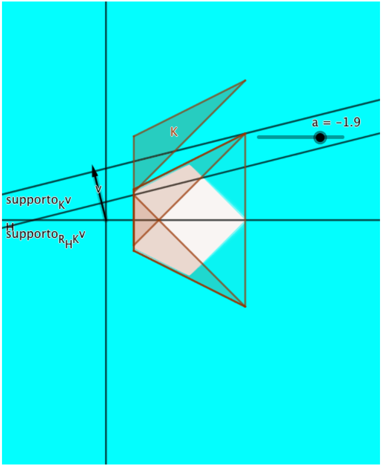

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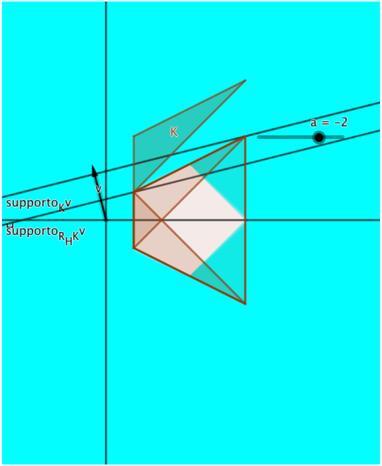

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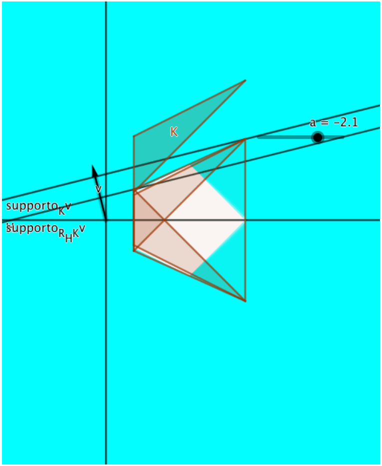

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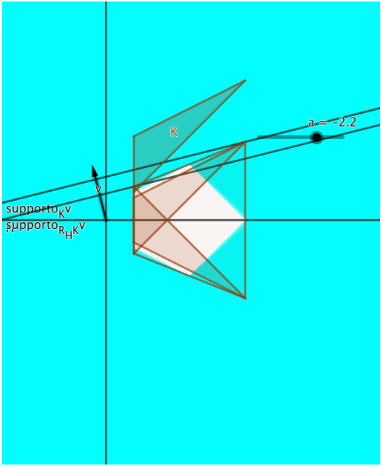

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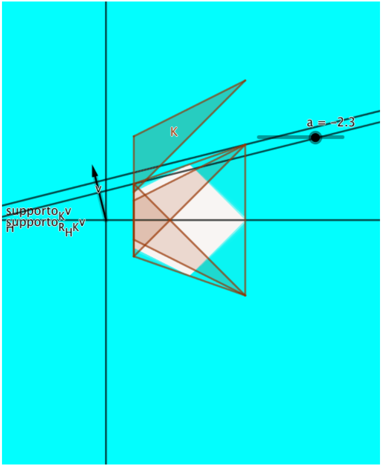

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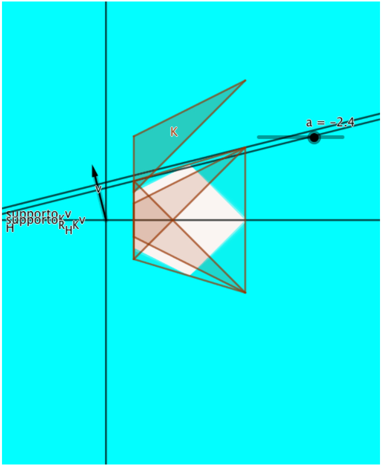

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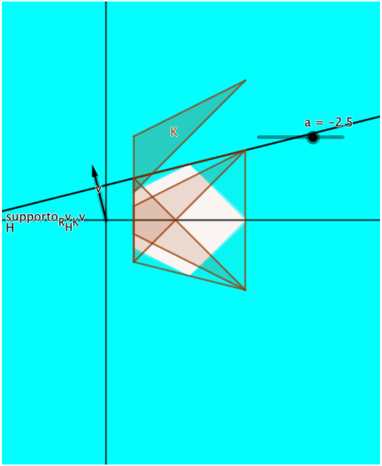

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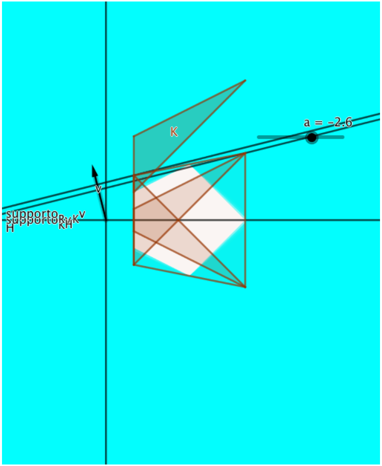

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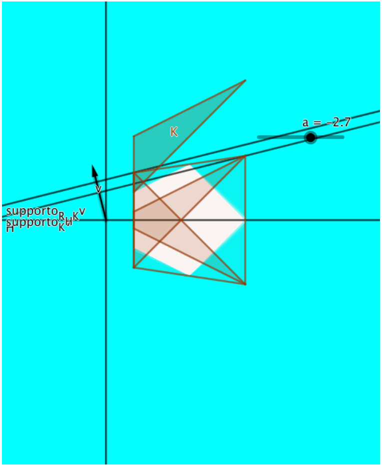

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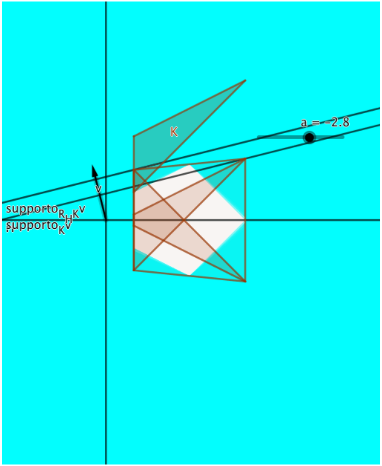

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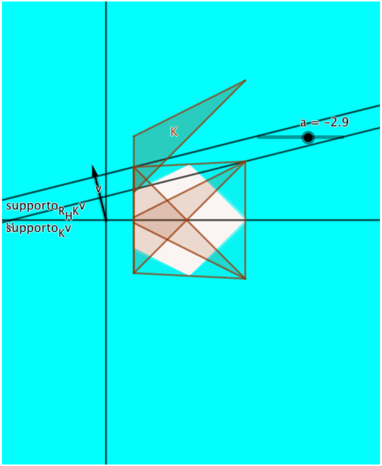

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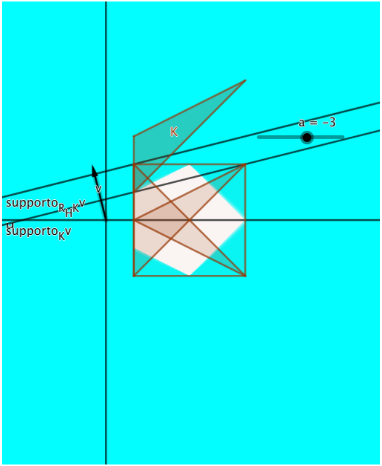

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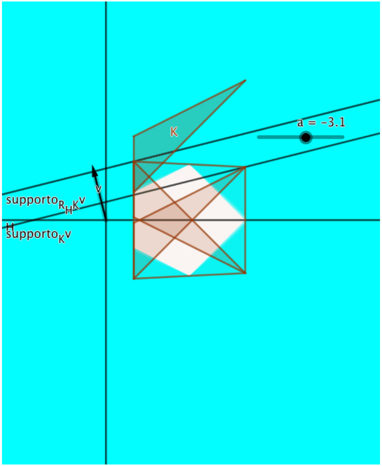

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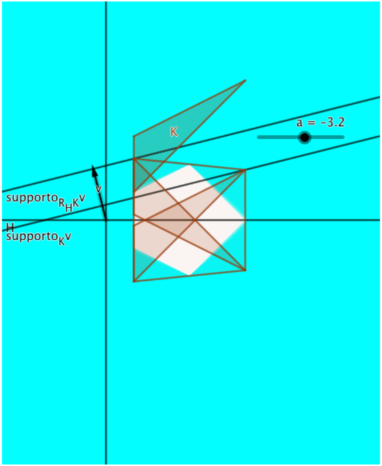

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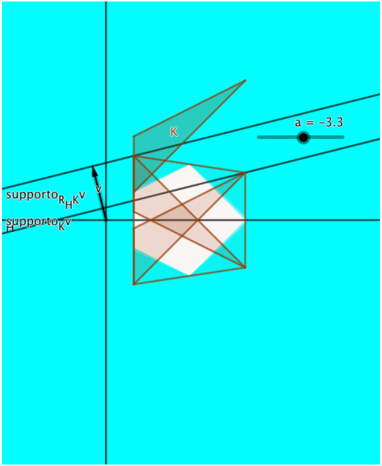

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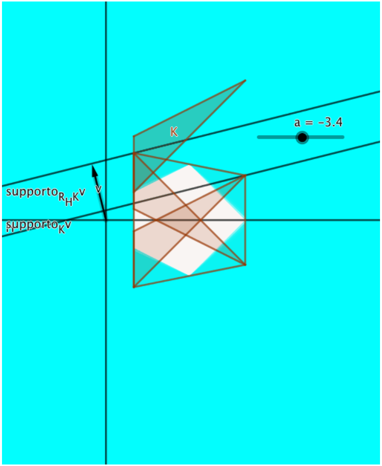

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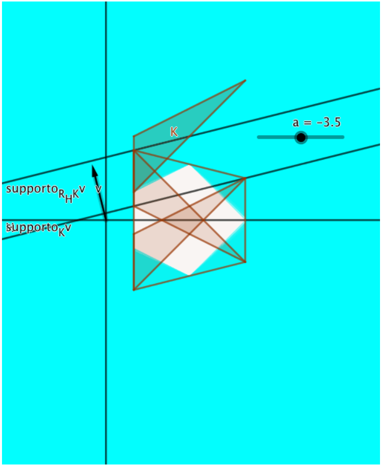

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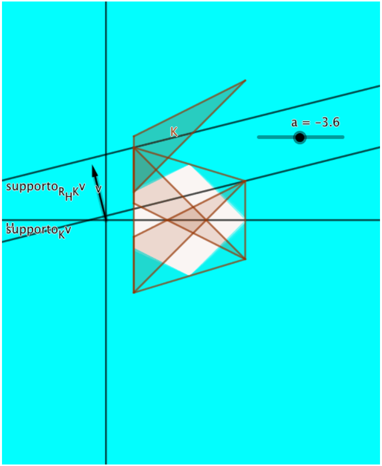

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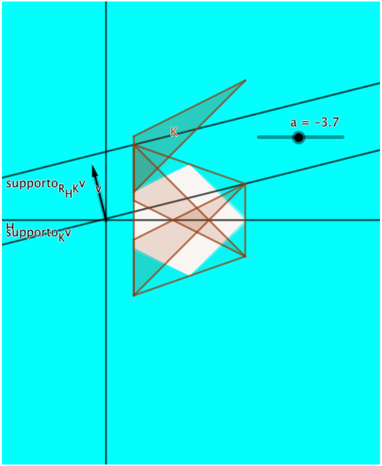

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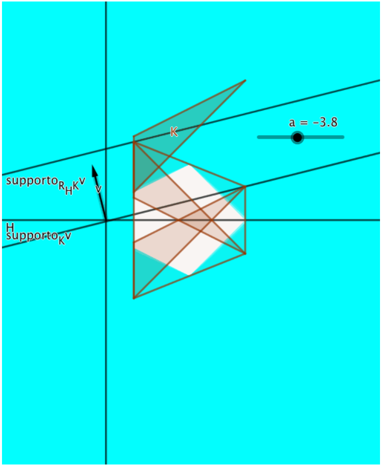

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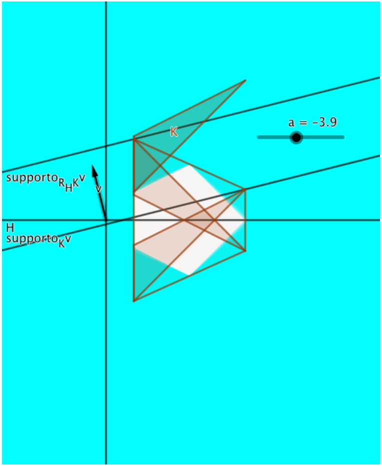

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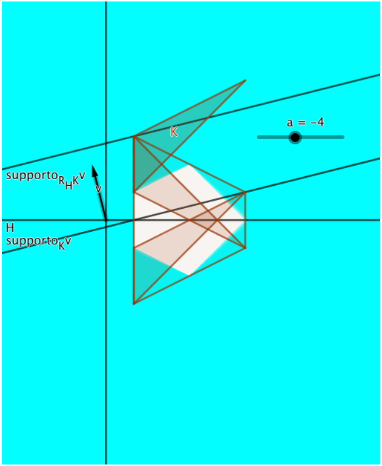

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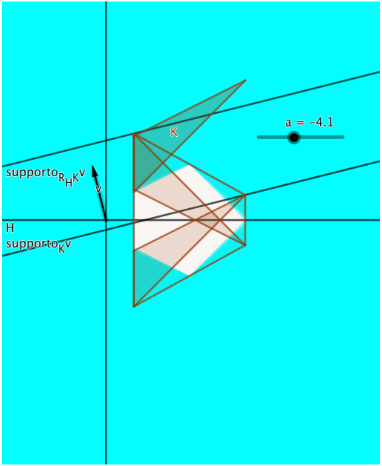

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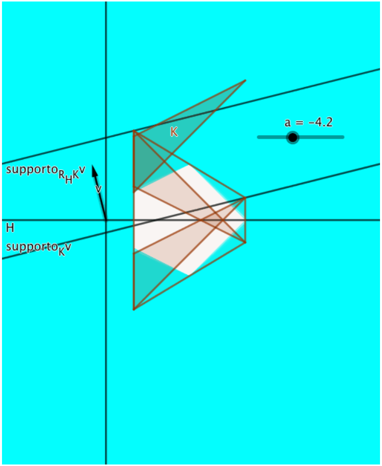

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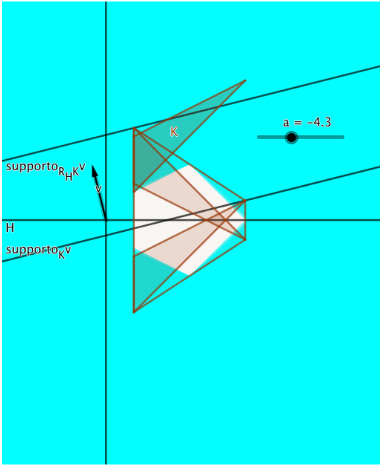

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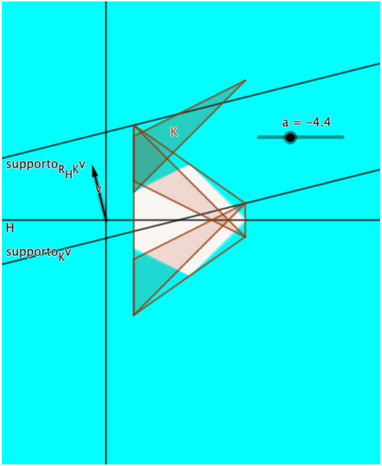

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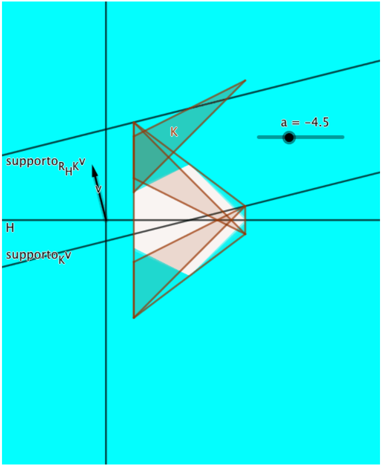

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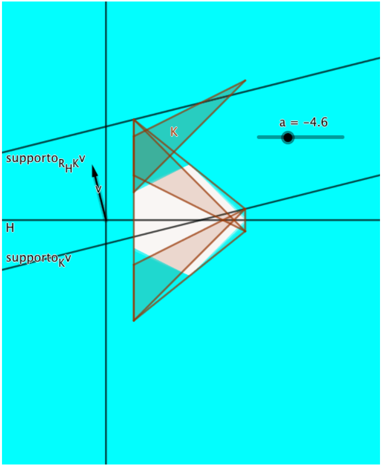

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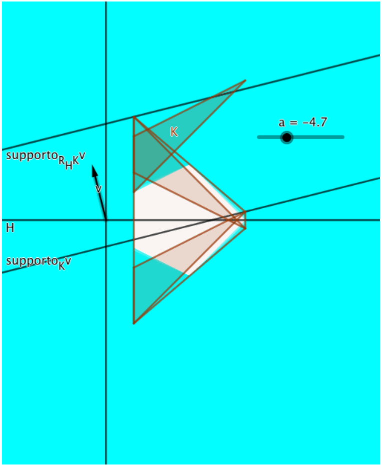

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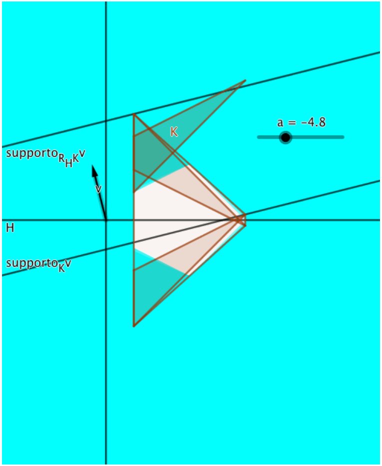

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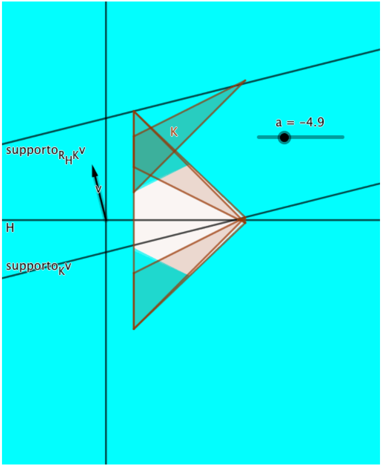

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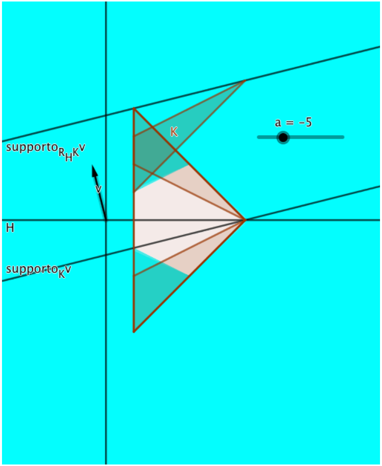

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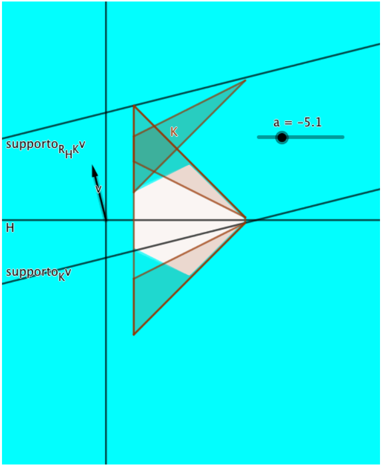

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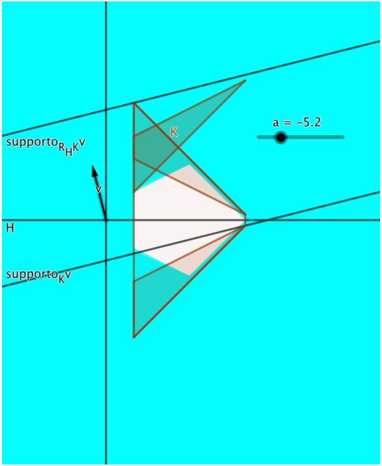

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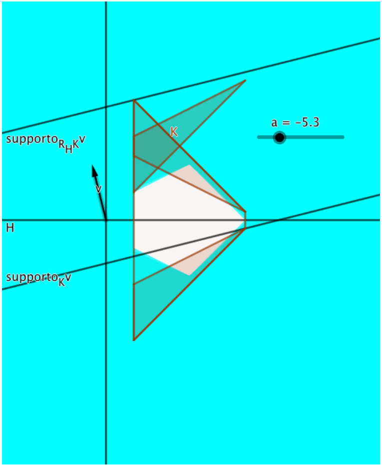

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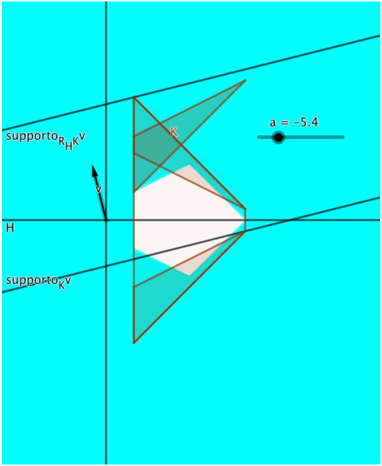

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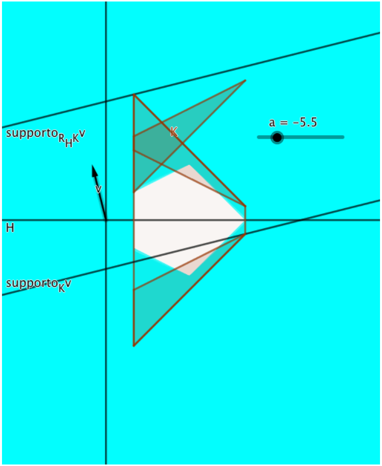

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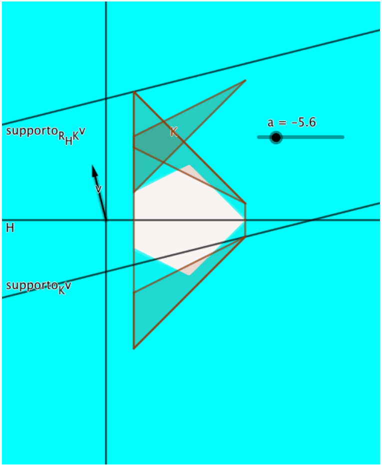

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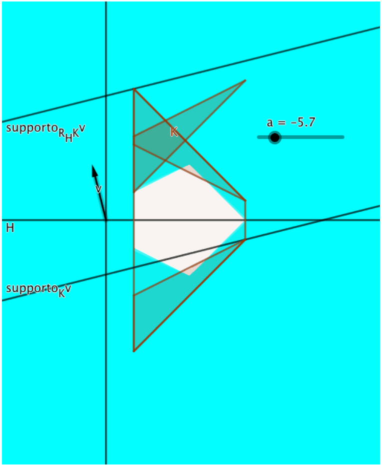

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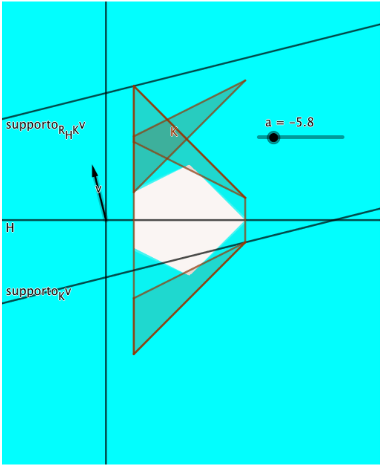

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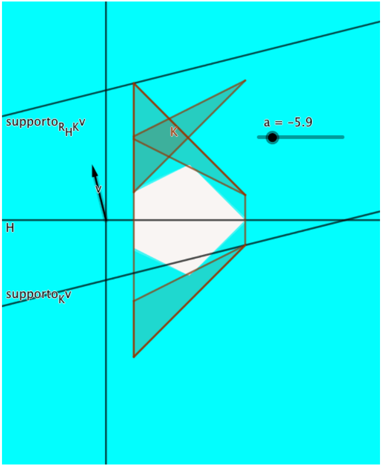

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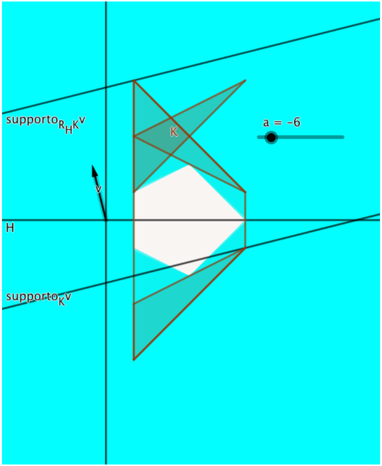

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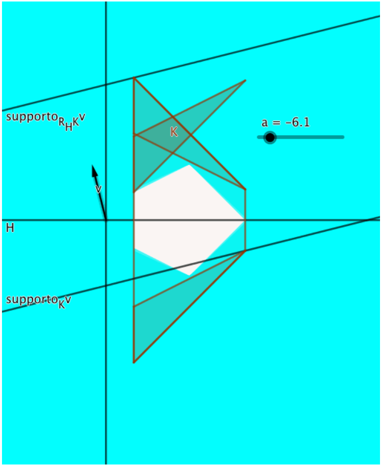

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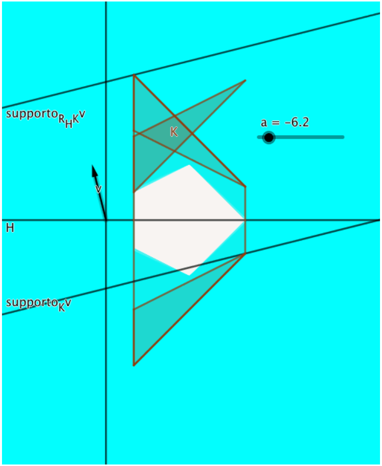

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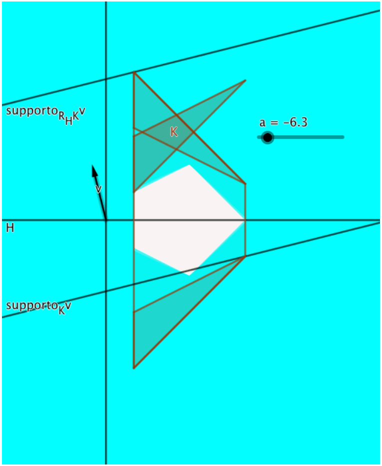

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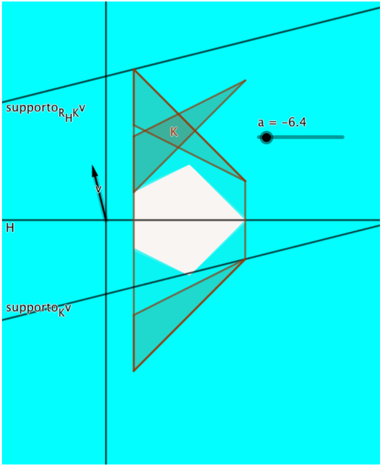

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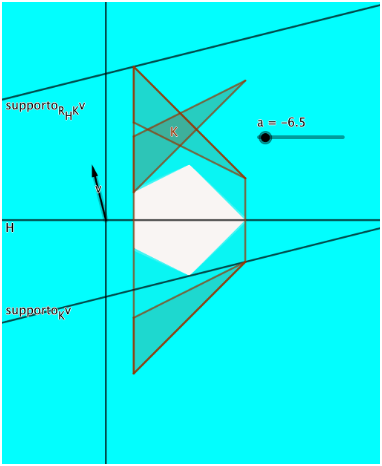

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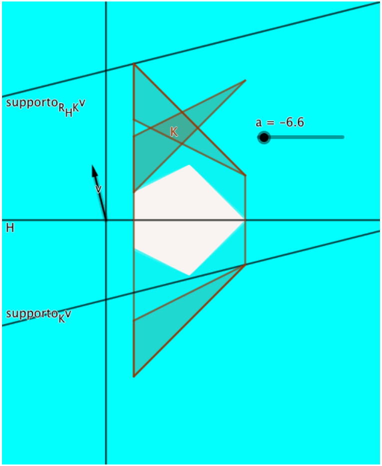

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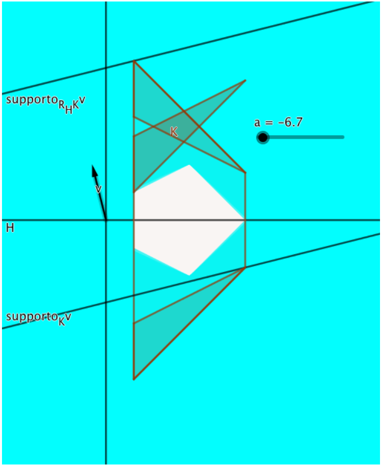

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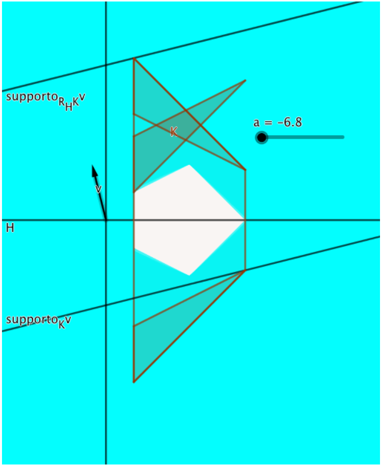

Let $v \in S^{n-1}$. For the support functions we have

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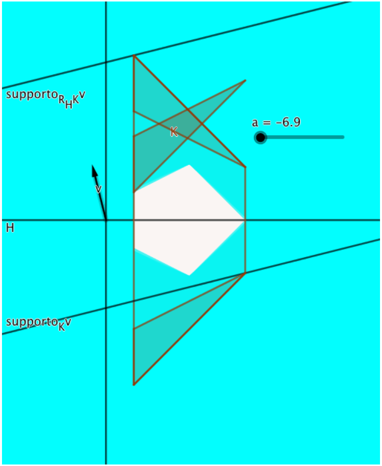

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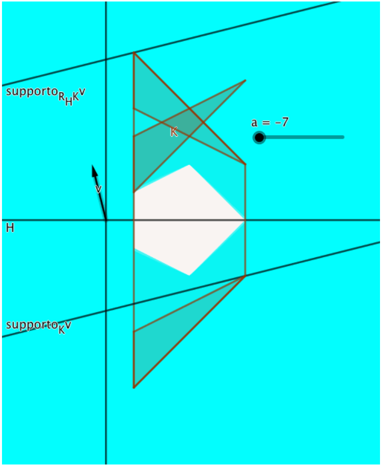

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Inclusions of general symm.

Corollary

Let $1 \leq i \leq n-1$ and $K \in \{\text{convex bodies}\}$. If $\diamondsuit_H$ is

1. monotonic,
2. invariant on H-symmetric sets and
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then

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Since this holds $\forall y$, we have proved that

$$\diamondsuit K \subset \bigcap_{y\in H^{\perp}} conv\left((K+y)\cup R_H(K+y)\right) = M_H K$$

Characterizations of Steiner and Minkowski symmetrizations

characterizations of Minkowski symmetrization

characterization 1

For every i and in the class $\{$convex bodies$\}$. Minkowski symmetrization is the only i-symmetrization which is

1. monotonic,
2. invariant on H-symmetric sets and
3. linear.

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4. mean width preserving.

- we do not have any example showing that in characterization 2 assumption 3 is really necessary.

characterizations of Steiner symmetrization

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Let $i = n - 1$ and let the class be $\{$convex bodies$\}$. Steiner symm. is the only i-symm. which is

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An open problem

Is there an (n-1)-symmetrization in {convex bodies} which is

1. monotonic,
2. invariant on H-symmetric sets,
3. and surface area preserving?

- Blaschke symm. preserves surface area but is not monotonic
- a partial answer is available in

C. Saroglou, On some problems concerning symmetrization operators, Forum Mathematicum 2019.

Convergence of iterates of symmetrizations to a ball

Let $\diamondsuit_H$ be Steiner or Minkowski symmetrization
It is known that there are sequences (H_m) of hyperplanes such that, for any choice of the convex body K, as $m \to \infty$

$$\left(\diamondsuit_{H_m} \diamondsuit_{H_{m-1}} \dots \diamondsuit_{H_1} K\right) \to \text{ball}.$$

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3 ingredients in this phenomenon:

- the choice of the symmetrization $\diamondsuit_H$
- the sequence (H_m) of subspaces (and, in particular, their dimension i)
- the class of subsets of $\mathbb{R}^n$ on which the $\diamondsuit_H$ acts

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We are interested in studying this process for different symmetrizations, set class and to better understand which sequences "round"

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There exists a convex body $K \subset \mathbb{R}^2$ and a sequence (H_m) of lines, dense in S^1, such that

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In $\mathbb{R}^n$, for each n, it is possible to rearrange any dense sequence (H_m) so that it "rounds" every convex body.

Bianchi, Klain, Lutwak, Yang and Zhang (2011)

Literature

- speed of convergence to ball (how many symmetrizations are needed to transform a convex body in R^n of volume 1 to one at ε distance from the ball of volume 1?): Bourgain, Lindestrauss, Milman, Klartag, Florentin and Segal
- results of probabilistic type : Mani-Levitska, Volčič, Van Shaftingen, Fortier e Burchard, Coupier e Davydov.

universal sequences

Coupier e Davydov (2014)

(H_m) is called an $\diamondsuit$-*universal sequence* in the set class $\mathcal{E}$ if

$$\forall K \in \mathcal{E}, \qquad \forall j \in \mathbb{N} \qquad \left(\diamondsuit_{H_m}\diamondsuit_{H_{m-1}}\dots\diamondsuit_{H_j}K\right) \to \text{ball},$$

(convergence to ball independently of starting index)

Universal sequences deserve this name

Theorem, Coupier and Davidov (2014)

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Is it more difficult to "round" compact sets?

"a compact set need not become convex"

There exists compact sets $C \subset \mathbb{R}^2$ and "meaningful" sequences (H_m) such that

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Theorem

Let $1 \leq i \leq n-1$ and let $\diamondsuit$ be Steiner, Minkowski or Schwarz symm. A sequence is $\diamondsuit$-universal in the class of {compact sets} if it is $\diamondsuit$-universal in the class of {convex bodies}

Explicit construction of universal sequences

"Alphabet" = finite set $\mathcal{F} = \{F_1, \ldots, F_p\}$ of i-dimensional subspaces in $\mathbb{R}^n$

Sequences built from a finite "alphabet"

Sequences (H_m) with the property that every their element belongs to $\mathcal{F}$

Example: $(H_m) = F_3, F_3, F_1, F_4, F_2, F_3, F_1, F_3, F_1, F_1, F_4, \ldots$

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These sequences are universal if the alphabet $\mathcal{F}$ has the following property:
The (reflection) symmetry w.r.t every F_j implies full radial symmetry.

This research contains also results regarding how to construct alphabets with this property.